

Credit: 4

SYLLABUS

Sets and Lebesgue measure:

Countable sets, uncountable sets, relation between the cardinality of a nonempty set and the cardinality of its power set, ring of set, Boolean ring, σ -ring, Boolean algebra and σ -algebra of sets, Set function. Outer measure, Measurable sets, Non measurable sets and Lebesgue measure, Example of non-measurable sets.

Measurable functions and Convergence theorem:

Measurable functions, Simple function, Step function, The Lebesgue integral of a bounded function over a set of finite measure, The integral of nonnegative measurable functions. The general Lebesgue integral, Fatou's lemma, Monotone convergence, dominated Convergence theorem, Dini's theorem, Convergence in measure. Differentiation of monotone functions, Functions of bounded variation, Differentiation of an integral, Absolute continuity, Convex functions. Measure spaces, Measurable functions, Integration, General convergence theorems.

L_p Space and Weierstrass approximation theorem:

The L_p space, Holder's inequality, Schwarz inequality, Reisz Fisher theorem, Theorem on Lebesgue integration, Weierstrass approximation theorem .

Signed measure and Product measure

Signed measures, Hahn decomposition theorem, Jordan decomposition theorem, Lebesgue decomposition theorem, The Radon-Nikodym Theorem., Product measures, Fubini's theorem, Relation between Riemann and Lebesgue.

REFERENCES:

1. G. De Barra (2013). *Measure theory and integration* (2nd edition), New age International Publisher,
2. H.L. Royden and P.M. Fitzpatrick (2010). *Real Analysis*, Fourth Edition, Pearson Publication.
3. S. J. Taylor (1973). *Introduction to Measure and Integration*, Cambridge University Press.