S-134 Roll No. …………………..

PHY–501

Mathematical Physics and Classical Mechanics

M. Sc. PHYSICS (MSCPHY-12/13/16/17)

First Year, Examination, 2018

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Show that the orthogonal property of Hermites polynomials is :

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x)\, dx = \begin{cases} 0 & m \neq n \\ 2^n n! \sqrt{n} & m = n \end{cases}$$

and find value the of :

$$\int_{-\infty}^{\infty} e^{-x^2} \left[H_2(x) \right]^2 dx .$$

2. (a) Find the complex form of the Fourier integral representation of :

$$f(x) = \begin{cases} e^{-kx}, & x > 0 \text{ and } k > 0 \\ 0, & \text{otherwise} \end{cases}$$

(b) Find the Laplace transform of 4 cosh 2*t* sin 4*t*.

3. (a) Define Hamiltonian. When is it equal to the total energy of the system ? Is this equality valid in general ? When is it constant of motion ?

(b) A particle of mass *m* moves in a conservation force field ? Write Lagrange's equation of motion in cylindrical co-ordinate.

4. Obtain an expression for the Lagrange's interpolation formula when the spacing between two successive points is contant.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (8) marks each. Learners are required to answer *four* (04) questions only.

1. Using the recurrence relation, show that :

$$4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$$

2. Find the solution of three-dimensional Laplace equation in Cartesian co-ordinates.

3. Show that $g_{\mu\nu}$ is a covariant tensor of the second rank.

4. Find the Christoffel's symbols corresponding to :

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

5. A simple pendulum is suspended from a massless spring of spring constant k which is confined to move along a vertical line. Set up Lagrange's equation for small oscillations.
6. Show that Poisson brackets are canonically invariant.
7. Explain the basic idea of numerical differentiation. Discuss with one example.
8. Discuss Newton-Cotes method for numerical integration. Obtain an expression for Trapezoidal rule.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this Section are compulsory.

1. Let $P_n(x)$ be Legendre polynomial of degree $n > 1$, then $\int_{-1}^{+1} (1 + x) P_n(x)\, dx$ is equal to :

   (a) 0

   (b) 1/(2*n* + 1)

   (c) 2/(2*n* + 1)

   (d) $n/(2n + 1)$

2. The value of the integral $\int x^2 J_1(x)\, dx$ is :

   (a) $x^2 J_1(x) + c$

   (b) $x^2 J_{-1}(x) + c$

   (c) $x^2 J_2(x) + c$

   (d) $x^2 J_{-2}(x) + c$

3. In the Fourier transform the value of $\int_0^\infty \frac{\sin ax}{x} dx$ is :

(a) π

(b) 2π

(c) $\frac{\pi}{2}$

(d) $\frac{\sqrt{\pi}}{2}$

4. The Laplace transform of the function

$$F(t) = \begin{cases} 1, 0 \le t < 2, \\ -1, 2 \le t \le 4, \end{cases} f(t+4) = f(t)$$

is given as :

(a) $\frac{1 - e^{-2s}}{s\left(1 + e^{-2s}\right)}$

(b) $\frac{1 + e^{-2s}}{s\left(1 + e^{-2s}\right)}$

(c) 0

(d) $\frac{s+1}{s-1}$

5. $A^{ijk}_{lm} B^{m}_{l}$ is a tensor of rank :

(a) 7

(b) 6

(c) 5

(d) 3

6. The Christoffel symbols of the first kind $[i, j, k]$ is :

(a) $\frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right]$

(b) $\frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k}\right]$

(c) $\frac{1}{2}\left[\frac{\partial g_{ik}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} - \frac{\partial g_{ij}}{\partial x^k}\right]$

(d) $\frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k}\right]$

7. "If ψ and ϕ are two integrals of the motion, their Poisson brackets likewise an integral of motion." This statement is true for :

(a) D' Alembert's principle

(b) Hamilton-Jacobi theory

(c) Poisson's theorem

(d) None of these

8. In the case of canonical transformation :

(a) The form of the Hamilton equations is preserved

(b) The form of Lagrange equation is preserved

(c) Hamilton's principle is satisfied in old as well as in the new co-ordinates

(d) The form of the Hamilton's equations may or may not be preserved.

9. For Lagrange's interpolation formula if $f(0.5) = 4.56$ and $f(0.8) = 5.07$, the value of $f(0.55)$ is :

   (a) 4.645

   (b) 6.326

   (c) 4.222

   (d) 5.326

10. Runge-Kutta method for second order first degree linear differential equation is :

    (a) $\frac{dy}{dx} = f\left(x, y, \frac{d^2y}{dx^2}\right)$

    (b) $\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right)$

    (c) $\frac{dy}{dx} = f\left(x, y, \frac{dy}{dx}\right)$

    (d) $\frac{d^2y}{dx^2} = f\left(x, y, \frac{dx}{dy}\right)$