S-740 Roll No. ……………………..

MAT–510

Mathematical Programming

M. Sc. MATHEMATICS (MSCMAT–12)

Second Year, Examination, 2018

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Solve the following Linear Programming Problem (L. P. P.) using revised Simplex method :

Max. :

$$z = 3x_1 + 6x_2 + 2x_3$$

S. t. :

$$3x_1 + 4x_2 + x_3 \leq 2$$

$$x_1 + 3x_2 + 2x_3 \leq 1$$

$$x_1, x_2, x_3 \geq 0$$

2. Obtain the necessary and sufficient conditions for the optimum solution of the following Non-linear Programming Problem (NLPP) :

Minimize :

$$z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

Subject to :

$$x_1 + x_2 + x_3 = 15$$

$$2x_1 - x_2 + 2x_3 = 20$$

$$x_1, x_2, 2x_3 \geq 0$$

3. Explain Bellman's optimality principle. Use Bellman's principle to divide a positive quantity '*b*' into *n* parts in such a way that their product is maximum.

4. Solve the following quadratic programming problem by Beale's method :

Max. :

$$f(x_1, x_2) = x_1 + x_2 - x_1^2 + x_1x_2 - 2x_2^2$$

Subject to :

$$2x_1 + x_2 \leq 1$$

$$x_1, x_2 \geq 0$$

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (8) marks each. Learners are required to answer *four* (04) questions only.

1. Show that $f(x) = x^2$ is a convex function.

2. Solve the following L. P. P. by Simplex method :

Max. :

$$z = 3x_1 + 5x_2 + 4x_3$$

S. t. :

$$2x_1 + 3x_2 \leq 8$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$2x_2 + 5x_3 \leq 10$$

and $x_1, x_2, x_3 \geq 0$.

3. Solve the following integer linear programming problem by branch and bound method :

Max. :

$$z = x_1 + x_2$$

S. t. :

$$3x_1 + 2x_2 \leq 12$$

$$x_2 \leq 2$$

$x_1, x_2 \geq 0$ and both are integers.

4. Distinguish between pure and mixed integer programming.

5. Solve the following non-linear programming problem graphically :

Max. :

$$f(x_1\ x_2) = x_1 + 2x_2$$

S. t. :

$$x_1^2 + x_2^2 \leq 1$$

$$2x_1 + x_2 \leq 2$$

and $x_1, x_2 \geq 0$.

6. What are the Kuhn-Tucker conditions and how are they important in the theory of non-linear programming ?

7. Solve the L. P. P. using dynamic programming :

Max. :

$$z = 3x_1 + 7x_2$$

S. t. :

$$x_1 + 4x_2 \le 8$$

$$x_2 \le 8$$

and $x_1, x_2 \ge 0$.

8. Write the quadratic form in matrix vector notation :

$$f(x) = x_1^2 - 2x_1x_2 + 4x_2^2$$

Also, determine the sign definiteness of the quadratic form :

$$\begin{bmatrix} 2 & 1 & 4 \\ 6 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

Choose the correct answer :

1. Branch and Bound method divides the feasible solution space into smaller parts by :

(a) branching

(b) bounding

(c) enumerating

(d) All of the above

2. A hyperplane is a :

(a) Open and convex set

(b) Close and convex set

(c) Non-convex set

(d) None of the above

3. Identify the incorrect statement : A quadratic form Q(X) is :

(a) Positive definite iff $Q(X) > 0$.

(b) Negative definite iff $Q(X) < 0$.

(c) Indefinite if $Q(X) > 0$ for some X and $Q(X) < 0$ for some other X.

(d) Positive definite as well as negative definite irrespective of sign of Q(X).

4. The graphical method of linear programming problem uses :

(a) Objective function equations

(b) Constraint equations

(c) Linear equations

(d) All of the above

5. If dual has an unbounded solution, primal has :

(a) no feasible solution

(b) unbounded solution

(c) feasible solution

(d) None of the above

Write True/False in the following statements :

6. Branch and Bound method is integer programming.
7. Quadratic programming problem is a convex programming problem.
8. The necessary condition will be sufficient to minimize a concave function.
9. The dual of the dual of the quadratic programming problem is the quadratic program itself.
10. In non-linear programming problem, the optimal solution always lies at a corner or edge of the feasible region.