S-700 Roll No. …………………..

MAT–506

Analysis and Advanced Calculus

M. Sc. MATHEMATICS (MSCMAT–12)

Second Year, Examination, 2018

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Consider the linear space of all *n*-tuples $x = (x_1, x_2, \ldots.., x_n)$ of scalars and define the norm by $\|x\|_\infty = \max \{|x_1|, |x_2|, \ldots., |x_n|\}$. If this space is denoted by l_∞^n, show that $\left(l_\infty^n, \|.\|_\infty\right)$ is a Banach space.
2. Define 'Hilbert Space' with an example. Show that 'The inner product in a Hilbert space is jointly continuous'.
3. Define 'Adjoint Operator'. Let T be an operator on a Hilbert space H, then $\exists$ a unique linear operator T^α on H s. t. :

$$(T_{x,y}) = (x, T^* y) \, \forall \, x, y \in H$$

where T^α is the adjoint operator of T.

4. State and prove Spectral theorem.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (8) marks each. Learners are required to answer *four* (04) questions only.

1. Show that 'every normal linear space is a metric space.'
2. Let T be a linear transformation from a normed linear space N into the normed linear space N', then prove that if T is continuous at the origin, then T is bounded.
3. Define 'Inner Product Space'. In a inner product space show that :

$$(x, \beta y + \gamma z) = \bar{\beta}(x, y) + \bar{\gamma}(x, z)$$

4. In any Hilbert space define orthogonal sets. Show that if :

$$S_1 \perp S_2 \Rightarrow S_1 \cap S_2 = \{0\}$$

5. Show that an orthonormal set S in a Hilbert space H is complete iff :

$$x \perp S \Rightarrow x = 0, \forall x \in H$$

6. An operator T on H is self-adjoint, then $(Tx, y) = (x, Ty) \forall x, y \in H$ are conversely.
7. If T is normal operator on a Hilbert space H, then eigen space of T are pairwise orthogonal.
8. Define 'Step function' and 'Regulated function'.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

Write whether the following statements are true *or* false :

1. If $x, y, z \in N$, N being a normed linear space, then $d(x + z, y + z) = d(x, y)$.
2. Every subsequence of $< x_n >$ converge weakly to *x*.
3. If *f* is a bounded linear functional on a complex normed space, then **F** is linear.

Fill in the blanks :

4. $(x, \beta y . \gamma z) = \ldots\ldots\ldots$
5. A .... inner product space is called a Hilbert space.
6. $(\alpha T)^* = \ldots\ldots\ldots$
7. (ST)* = ...
8. If T is a normal operator on a Hilbert space H, then eigen spaces of T are pairwise ....
9. The limit of a convergent sequence is ....
10. (*x*, 0) = ...