S-676 Roll No. ……………………..

MAT–501

Advanced Algebra

M. Sc. MATHEMATICS (MSCMAT-12)

First Year, Examination, 2018

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Define 'Direct product of two groups'. Let G_i $(1 \le i \le n)$ be n groups and G is the external direct product of these groups, than each $g \in G$ can be written uniquely as product of elements from G_1, G_2, ..., G_n.

2. Let G be a finite group, then $o(G) = \sum_{a \in D} \frac{o(G)}{o\, N(a)} = \sum_{i=1}^{n} \frac{o(G)}{o[N(a_i)]}$, where D be a set of distinct elements $a_1, a_2, ..., a_n$ taken from each of the conjugate classes of G.

3. Define 'Real inner product space' and 'Norm of a Vector'. Let V be an inner product space. Then for arbitrary vectors $u, v \in$ V we have $|< u, v >| \leq \|u\| \|v\|$.

4. Let V be a finite dimensional inner product space and W be its subspace. Then V is direct sum of W and $W^{\perp}$. Also show that dim $W^{\perp} = \dim V - \dim W$.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (8) marks each. Learners are required to answer *four* (04) questions only.

1. Let G be a group. H and K are two subgroups of G such that H and K are normal in G and H $\cap$ K = $\{e\}$, then HK is the internal direct product of H and K.

2. Define centre of a group. Show that centre of a group is a normal subgroup of the group.

3. Define solvable group. Show that every finite abelian group is solvable.

4. Define 'Euclidean ring'. Show that 'Every field is a Euclidean ring'.

5. Define Rank and nullity of a linear transformation. Let $t : V_2(R) \rightarrow V_3(R)$ defined by $t(a, b) = (a + b, a - b, b)$. Find rank and nullity of above linear transformation.

6. If K is a finite field extension of a field F and L is a finite field extension of K, then L is a finite field extension of F and [L : F] = [L : K] [K : F].

7. Let $t : R^3 \to R^3$ be a linear transformation such that $t(a, b, c) = (3a + c, -2a + b, -a + 2b + 4c)$. What is the matrix of t in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$, where :

$$\alpha_1 = (1, 0, 1)$$

$$\alpha_2 = (-1, 2, 1)$$

$$\alpha_3 = (2, 1, 1)$$

8. Let V be a finite dimensional vector space over a field F. Then the set of all eigen vectors corresponding to an eigen value λ of a linear transformation $t : V \to V'$ by adjoining zero vector to it, is a subspace of V.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

Indicate whether the following statements are True *or* False :

1. Let G_1 and G_2 be groups, then $G_1 \times G_2 \cong G_2 \times G_1$. (True/False)
2. N (*a*) is normal subgroup of group. (True/False)
3. An infinite abelian group does not have a composition series. (True/False)
4. Every principal ideal domain is an 'Euclidean ring'. (True/False)
5. Let $t : V \to V'$ be a linear transformation. Then ker (*t*) is a vector subspace of V. (True/False)

Fill in the blanks :

6. Let V and V' be vector spaces over the same field F and $t : V \rightarrow V'$ be a linear transformation, then rank (t) + nullity (t) = ...

7. Every field of characteristic zero is ....

8. For any matrix A over a field F rank $\left(A^{T}\right)$ = ...

9. A square matrix A of order n is invertible if rank (A) = ...

10. A is a square non-singular matrix. then deg (A^{-1}) = ...