Roll No. ....

PHY–501

Mathematical Physics and Classical Mechanics

M. Sc. PHYSICS (MSCPHY–12/13/16)

First Year, Examination, 2017

Time : 3 Hours **Max. Marks : 70**

Note : This paper is of **seventy (70)** marks containing **three (03)** sections A, B and C. Attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. Using Rodrigue's formula prove that the following :

(a) $\int_{-1}^{+1} P_0(x)\, dx = 2$

(b) $\int_{-1}^{+1} P_n(x)\, dx = 0 \ (n \neq 0)$

2. (a) Find the Fourier transform of F(t) defined by :

$$F(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases}$$

and hence evaluate :

$$\int_0^\infty \frac{\sin \omega t}{\omega}\, d\omega$$

(b) Find the Laplace transform of :

$$F(t) = \begin{cases} 0 & 0 < t < \frac{1}{2} \\ \sin \pi t & t > \frac{1}{2} \end{cases}$$

3. (a) If A_i is a covariant vector, show that $\frac{\partial A_i}{\partial x^j}$ is not a tensor but $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ is a second order covariant tensor in the generalized co-ordinate system.

(b) Obtain the Lagrange's equation of motion for simple pendulum.

4. (a) Explain Hamilton's principle and discuss their one application.

(b) Derive Newton's formula for towards interpolation and explain the assumptions for its validity.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (05) marks each. Learners are required to answer *six* (06) questions only.

1. Show that the when n is a positive integer :

$$J_n(x) = (-1)^n J_n(x)$$

2. For Hermite polynomial show that :

$$H_n(-x) = (-1)^n H_n(x)$$

3. Find the finite Fourier sine transform of $e^{\alpha t}$ in the interval $(0, \pi)$.
4. Show that Kronecker delta is a mixed tensor of rank two.
5. Write the covariant derivative of a mixed tensor of rank 2.
6. Discuss in brief Hamilton-Jacobi equations.
7. Show that Poisson brackets are invariant under a canonical transformation are :

$$[x, y]_{q,p} = [x, y]_{\mathrm{Q,P}}$$

8. State the assumptions for the validity of Netwon's forward and backward interpolation formulas.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

1. The integral $\int_0^{\pi} \mathrm{P}_n(\cos\theta)\sin 2\theta\, d\theta$, $n > 1$ when $\mathrm{P}_n(x)$ is the Legendre polynomial of degree $n > 1$, then equal to :

   (a) 1

   (b) $\frac{1}{2}$

   (c) 0

   (d) 2

2. For Bessel's function $J_{1/2}$ is given by :

(a) $\sqrt{\frac{2\pi}{x}}\sin x$

(b) $\sqrt{\frac{2\pi}{x}}\cos x$

(c) $\sqrt{\frac{\pi}{2x}}\cos x$

(d) $\sqrt{\frac{\pi}{\pi x}}\sin x$

3. The Fourier sine integral formula for $f(x)$ is :

(a) $\frac{2}{\pi}\int_0^\infty\int_0^\infty f(t)\ \sin ut \sin ux\, du\, dt$

(b) $\frac{1}{2\pi}\int_0^\infty\int_0^\infty f(t)\ \cos ut \cos ux\, du\, dt$

(c) $\frac{1}{3\pi}\int_0^\infty\int_0^\infty f(t)\ \cos ut \sin ux\, du\, dt$

(d) $\frac{2}{\pi}\int_0^\infty\int_0^\infty f(t)\ \sin ut \cos ux\, du\, dt$

4. Laplace transform of t^3e^{-3t} is :

(a) $\frac{7}{(s+4)^3}$

(b) $\frac{5}{(s+3)^3}$

(c) $\frac{6}{(s+3)^4}$

(d) $\frac{7}{(s+6)^3}$

5. If A^{μ} and B_{μ} are components of contravariant and covariant tensors what is the nature of the quantity $A^{\mu}B_{\mu}$?

(a) Mixed tensor of rank 1

(b) A covariant tensor of rank 2

(c) A mixed tensor of rank 2

(d) Rank zero scale

6. $A^{ijk}_{lm}\ B^{m}_{l}$ is tensor of rank :

(a) 3

(b) 4

(c) 5

(d) 6

7. The equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial \dot{q}_j} = 0,$ where L = T – V :

(a) Lagrangian equation for conservative system

(b) Lagrangian equation for non-conservative system

(c) Equation of motion of hormonic oscillator

(d) None of the above

8. In the case of canonical transformation :

(a) The form of the Hamilton equations is preserved.

(b) The form of Lagrange equations is preserved.

(c) Hamilton's principle is satistfied in old as well as in the new co-ordinates.

(d) The form of the Hamilton's equations may or may not be preserved.

9. The differential form of Hermite polynomials is represented by :

(a) $H_n(x) = \frac{d^n}{dx^n}(e^{-x^2})$

(b) $H_n(x) = \frac{1}{n!}\frac{d^n}{dx^n}(e^{x^2})$

(c) $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$

(d) None of the above

10. Laplace transform of $e^{-2t} \sin 4t$ is :

(a) $\frac{2}{s^2 + 4s + 20}$

(b) $\frac{s - 2}{s^2 + 4s + 20}$

(c) $\frac{s - 4}{s^2 + 4s + 20}$

(d) $\frac{4}{s^2 + 4s + 20}$