Roll No. …………………..

MAT–510

Mathematical Programming

M. Sc. MATHEMATICS (MSCMAT–12)

Second Year, Examination, 2017

Time : 3 Hours **Max. Marks : 60**

Note : This paper is of **sixty (60)** marks containing **three (03)** sections A, B and C. Attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. Solve the following non-linear programming problem using the method of Lagrangian multipliers :

   Max. :

$$z = 6x_1 + 8x_2 - x_1^2 - x_2^2$$

   Subject to :

$$4x_1 + 3x_2 = 16$$

$$3x_1 + 5x_2 = 5$$

   and $x_1, x_2 \geq 0$.

2. By deriving the necessary Kuhn-Tucker conditions and using Wolf's method, solve the following quadratic programming problem.

   Max. :

   $$z = 2x_1 + x_2 - x_1^2$$

   Subject to :

   $$2x_1 + 3x_2 \leq 6$$

   $$2x_1 + x_2 \leq 4$$

   and $x_1, x_2 \geq 0$.

3. Define Duality. Derive the dual of the quadratic program :

   Min. :

   $$\left(\mathrm{C}^{\mathrm{T}} x + \frac{1}{2} x^{\mathrm{T}} \mathrm{C} x \right)$$

   Subject to :

   $$\mathrm{A}x \geq b$$

   where A is $m \times n$ real matrix and G is $m \times n$ symmetric matrix.

4. Use the revised Simplex method to solve the linear programming problem :

   Min. :

   $$z = -2x_1 - x_2$$

   S. t. :

   $$x_1 + x_2 \geq 2$$

   $$x_1 + x_2 \leq 4$$

   and $x_1, x_2 \geq 0$.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (5) marks each. Learners are required to answer *four* (04) questions only.

1. State the features of Dynamic programming problem.
2. Define convexity. Check whether the following function is a convex function or not :

$$f(x, y) = x^2 - 3xy + y^2$$

3. Solve the integer programming problem by branch and bound method :

   Max. :

$$z = 7x_1 + 9x_2$$

   Subject to :

$$-x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

   and $0 \leq x_1, x_2 \leq 7$.

4. What are separable programming function ? Explain with suitable example.
5. Solve by Kuhn-Tucker condition :

   Max. :

$$z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$$

   Subject to :

$$2x_1 + x_2 \leq 5$$

   and $x_1, x_2 \geq 0$.

6. Let $S \subset R^n$ be a convex set, H a supporting hyperplane of S and T = $S \cap H$, prove that every extreme point of T is an extreme point of S.
7. Write the algorithm for the solution of a integer programming problem by Gomory method.
8. Use dynamic programming to solve the linear programming problem :

   Max. :
   $$z = x_1 + 9x_2$$
   S. t. :
   $$2x_1 + x_2 \leq 25$$
   $$x_2 \leq 11$$
   and $x_1, x_2 \geq 0$.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

1. (A) Write True/False in the following questions :
   (i) Every local minimum of the convex program is a global minimum.
   (ii) Branch and bound methods are used to solved both pure and mixed integer linear programming problem.
   (iii) The optimum solution of a non-linear programming problem is at an extreme point of the feasible region.

(iv) The Hessian matrix used for solving non-linear programming problem is defined as the square matrix of second order partial derivatives.

(v) In quadratic programing, only the objective function is non-linear while all the ocnstraints are linear functions.

(B) Fill in the blanks in the following questions :

(i) A function $f\ \ x_1, x_2, ..., x_n$ is said to be separable if

$$f\ \ x_1, x_2, ..., x_n\ \ = ...$$

(ii) .... is used for getting the stationary points of the constrained non-linear programming problem.

(iii) Dynamic programming is a Mathematical technique dealing with the optimization of ....

(iv) In convex set, the line joining two points x_1 and x_2 in the set form is written as ....

(v) The Lagrangian multipliers of the programming problem :

Min (– x^2)

Subject to $0 \le x \le 1$

is ....