Roll No. ……..

MAT–507

Viscous Fluid Dynamics

M. Sc. Mathematics (MSCMAT–12)

Second Year, Examination, 2017

Time : 3 Hours **Max. Marks : 60**

Note : This paper is of **sixty (60)** marks containing **three (03)** sections A, B and C. Learners are required to attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. Derive the equation of continuity for perfect fluid in cylindrical coordinate system.
2. Show that the state of stress at a point is completely know if the nine components of stress tensor at that point are known.
3. Determine the dissipation of energy, which is dissipated in a viscous liquid in motion on account of the internal friction in Cartesian coordinate system.
4. Write short notes on the following :
   (a) Boundary layer thickness
   (b) Displacement thickness
   (c) Momentum thickness
   (d) Energy thickness

Section–B
(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (05) marks each. Learners are required to answer *four* (04) questions only.

1. Discuss the main limitations of Navier-Stokes equations.
2. The stress matrix at a point is given by

$$\begin{bmatrix} 7 & -5 & 0 \\ -5 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Determine the stress vector on the plane passing through P and having for its equation $\frac{x}{4} + \frac{y}{2} + \frac{z}{6} = 1$.

3. Oil is filled between two concentric rotating cylinders with radii 5 inch and 11/2 inch. Assume that $\mu = 0.005$ lbf-sec/ft^3. The inner cylinder rotates at a speed of 5 r. p. m., while the outer cylinder is at rest. Find the stress at the wall of the inner cylinder.
4. Determine the principal stresses and principal axes of the state of stress given by the stress tensor $T_{ij} = \alpha(\lambda_i \lambda'_j + \lambda'_i \lambda_j)$, where α is scalar and λ_i, λ'_i are unit vectors.
5. Show that for an incompressible steady flow with constant viscosity, the velocity components :

$$u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1 - \frac{y}{h}\right); v = w - 0$$

satisfy the equation of motion, when the body force is neglected. Here $h, U, \frac{dp}{dx}$ are constants and $p = p(x)$.

6. Discuss the main limitations of non-viscous fluid dynamics.
7. Explain plane Couette flow.
8. Explain the following :
   (a) Reynolds' number
   (b) Mach Number

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

1. The study the fluid, if we select any fixed point in space, occupied by space and study the changes which take place, the method is called :
   (i) Eulerian
   (ii) Lagrangian
   (iii) Newtonian
   (iii) Gaussian
2. A curve drawn in the fluid so that its tangent at each point is the direction of motion at that point. It is called :
   (i) path line
   (ii) stream line
   (iii) boundary surface
   (iv) None of these

3. A curve, along which a particular fluid particle travels during its motion, is called :
   (i) path line
   (ii) stream line
   (iii) boundary surface
   (iv) None of these

4. The stress at a point is a tensor of order :
   (i) one
   (ii) three
   (iii) any integer
   (iv) two

5. The dimensions of the coefficient of viscosity is :
   (i) MLT
   (ii) MLT^{-1}
   (iii) $ML^{-1}T^{-1}$
   (iv) $ML^{-1}T$

6. If we drawn the stream lines from each point of a closed curve in the fluid, we obtain :
   (i) stream line
   (ii) stream filament
   (iii) path tube
   (iv) None of these

7. Velocity (q) and velocity potential (ϕ) are connected as, $q = -$ grad ϕ . Here negative sign ensures that :
   (i) flow takes place from lower to higher potential

(ii) flow takes place from higher to lower potential

(iii) negative sign is insignificant

(iv) None of these

8. A curve drawn in the fluid such that the tangent to it at every point is in the direction of the vorticity vector (Ω) is called :

(i) streak line

(ii) path line

(iii) vortex line

(iv) stream line

9. The body forces are usually expressed as the force per unit ..... .

(i) area

(ii) volume

(iii) density

(iv) none of these

10. Boundary layer theory was formulated by :

(i) Prandtl

(ii) Reynolds

(iii) Navier

(iv) Stokes