Roll No. …………………..

MAT–506

Analysis and Advanced Calculus

M. Sc. Mathematics (MSCMAT–12)

Second Year, Examination, 2017

Time : 3 Hours **Max. Marks : 60**

Note : This paper is of **sixty (60)** marks containing **three (03)** sections A, B and C. Attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. Discuss convergence in Normed Linear space with an example. Show that the limit of a convergent sequence is unique.
2. Show that if B and B' are Banach spaces and T is a linear transformation of B into B', then T is continuous $\Leftrightarrow$ its graph is closed.
3. Define orthonormal sets. If $\{e_i\}$ is an orthonormal set in a Hilbert space H, then $\sum | x, e_i |^2 \leq \| x \|^2, \ \forall x \in \mathrm{H}$.

4. Let f be a continuous function on a compact interval $[a, b]$ of R into a Banach space X over K. Let F be the function $t \to \int_a^t f$ on $[a, b]$ into x. Let g be any differentiable function on $[a, b]$ into X such that $\mathrm{D}g = f$. Then F is differentiable, $\mathrm{D}f = f$ and

$$\int_a^b f = \mathrm{F}(b) = \mathrm{F}(a)$$

$$= g(b) - g(a).$$

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (05) marks each. Learners are required to answer *four* (04) questions only.

1. If N be a normed linear space with the norm $\|\cdot\|$, then the mapping $f : \mathrm{N} \to \mathrm{R}$ s. t. $f(x) = \|x\|$ is continuous. In other words, the norm $\|.\|$ on N is a continuous function.

2. Show that linear space C (complex) is normed linear space under the norm $\|x\| = |x|$, $x \in \mathrm{C}$, also show that above space is complete and hence Banach space.

3. The inner product in a Hilbert space is jointly continuous i. e. if $x_n \to x$, and $y_n \to y$, then $(x_n, y_n) \to (x, y)$ as $n \to \infty$.

4. If S is a non-empty subset of a Hilbert space H, then $S^{\perp}$ is a closed linear subspace of H and hence a Hilbert space.
5. If T is an arbitrary operator on Hilbert space H, then T = 0 iff $(Tx, y) = 0, \forall\, x, y \in H$.
6. If T_1 and T_2 are normal operators on H with the property that either commutes with adjoint of the other, then show that $T_1 + T_2$ is normal.
7. For a Hilbert space H, define eigenvalue, eigenvector and spectrum of T.
8. Define directional derivative for a Banach space.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

Fill in the blanks :

1. On a finite dimensional linear space X, all norms are ....
2. In any inner product space property $\overline{(x, y)} = (y, x)$ is called ....
3. A complete inner product space is called a ....
4. If $TT^* = T^{\alpha}T$, where T^{α} is adjoint of T in Hilbert space then T is called ....
5. The derivative of the constant function $f : V \to y$ is ....

Write T for True and F for False :

6. In a seminormed space

$$\| x \| = 0 \Rightarrow x = 0$$

7. Two norms $\| \cdot \|_1, \| \cdot \|_2$ defined on a normed space N are equivalent iff $\exists$ positive real numbers a and b s. t. :

$$a \| x \|_1 \le \| x \|_2 \ge b \| x_1 \| \quad \forall \ x \in \mathrm{N}$$

8. If x and y are any two orthogonal vectors in a Hilbert space H, then :

$$\| x + y \|^2 = \| x - y \|^2 = \| x \|^2 + \| y \|^2$$

9. If T is a normal operator and α is a scalar, then αT is normal.

10. If f is continuous, then f is regulated.