Roll No. …………………..

MAT–504

Differential Geometry and Tensors

M. Sc. MATHEMATICS (MSCMAT–12)

First Year, Examination, 2017

Time : 3 Hours **Max. Marks : 60**

Note : This paper is of **sixty (60)** marks containing **three (03)** sections A, B and C. Learners are required to attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. Find $f(u)$ so that the curve :

$$x = a\cos u$$
$$y = a\sin u$$
$$z = f(u)$$

determine a plane curve.

2. Obtain the differential equation of the lines of curvature through a point out the surface :

$$F(x, y, z) = 0$$

3. If the first, second and third fundamental forms are denoted by I, II and III respectively, then prove that :

$$KI - 2\mu\, II + III = 0$$

where K is the Gaussian curvature and μ is the mean curvature.

4. Obtain the differential equation of a geodesic in n-dimensional Riemannian space.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (05) marks each. Learners are required to answer *four* (04) questions only.

1. If a curve lies on a sphere show that ρ and σ are related by :

$$\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$$

2. Prove that the distance between corresponding points of two involutes is constant.

3. Find a unit normal vector to the surface :

$$2xz^2 - 3xy - 4x = 7$$

at the point $(1, -1, 2)$.

4. Show that if L, M and N vanish everywhere on a surface, then the surface is part of a plane.

5. Show that the curves :

$$du^2 - (u^2 + a^2)\, dv^2 = 0$$

form an orthogonal system on the right helicoid :

$$r = (u \cos v, u \sin v, av)$$

6. Prove that the surface given by :

$$e^z \cos x = \cos y$$

is a minimal surface.

7. Show that $g_{ij} dx^i dx^j$ is an invariant.

8. If Γ^{i}_{jk} denote Chirstoffel symbol of second kind, prove that :

$$\Gamma^{i}_{ij} = \frac{\partial \log \sqrt{g}}{\partial x^{j}}$$

where $g = \left| g_{ij} \right|$.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

1. The magnitude of the vector $\frac{dr}{ds}$ is ....
2. The plane spanned by the tangent vector *t* and the binormal *b* is called ....
3. If the curve is helix, the ratio of curvature and torsion is ....
4. The spherical image of a straight line is ....
5. If the Gaussian curvature is positive at a point, the point is called .... point.
6. A surface generated by the motion of one parameter family of straight lines is called ....
7. A covariant tensor of order two has .... components in *n*-dimensional space.

8. If $A^i, i = 1, 2, ..., n$ are functions of co-ordinates $(x^1, x^2, ... x^n)$ satisfying :

$$\overline{A}^i = A^j \frac{\partial \overline{x}^i}{\partial x^j}$$

then A^i are components of a .... vector.

9. The Christoffel symbols are .... tensor quantities.

10. The order of covariant derivative of an rth ranked tensor is equal to ....