Roll No. ……..

MAT–503

Differential Equations, Calculus of Variations and Special Functions

M. Sc. MATHEMATICS (MSCMAT–12)

First Year, Examination, 2017

Time : 3 Hours **Max. Marks : 60**

Note : This paper is of **sixty (60)** marks containing **three (03)** sections A, B and C. Learners are required to attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. Solve :

$$s - t = \frac{x}{y^2}$$

2. Find the extremals of the functional :

$$\int_0^{\pi} (4y \cos x + y'^2 - y^2) dx$$

that satisfy the given boundary conditions $y(0) = y(\pi) = 0$.

3. Prove that :

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{(2n+1)} & \text{if } m = n \end{cases}$$

4. Using the method of separation of variables, solve :

$$\frac{\partial u}{\partial x} = 2\left(\frac{\partial u}{\partial t}\right) + u$$

where $u(x, 0) = 6\, e^{-3x}$.

Section–B
(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (05) marks each. Learners are required to answer *four* (04) questions only.

1. Prove that :

$$J_{-1/2}(x) = \sqrt{\left(\frac{2}{\pi x}\right)}.\sin x$$

2. Prove that in Laguerre polynomial $\lfloor n(0) = 1$.

3. Show that :

$$\frac{d}{dx} F(\alpha, \beta; \gamma; x) = \frac{\alpha\beta}{\gamma} F(\alpha + 1, \beta + 1; \gamma + 1; x)$$

4. Solve :

$$(x^2 + y^2 + z^2)\, dx - 2xy\, dy - 2xz\, dz = 0$$

5. Solve boundary value problem :

$$\frac{\partial u}{\partial x} = 4\left(\frac{\partial u}{\partial y}\right),$$

if $u(0, y) = 8\, e^{-3y}$.

6. Find the extremal of the functional $\int_1^3 (3x - y)y\,dx$ that satisfy the boundary conditions $y(1) = 1$, $y(3) = \frac{9}{2}$.

7. Prove that :

$$H_n'(x) = 2x\,H_n(x) - H_{n+1}(x)$$

8. Solve :

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z(x + y)}$$

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

1. $P_n(-1)$ = ...
2. $Q_n' - x\,Q_{n-1}'$ = ...
3. $\lceil 1$ = ...
4. $J_{1/2}(x)$ = ...
5. $H_1(x)$ = ...
6. $F(\alpha, \beta; \gamma; 1)$ = ...
7. $\lim_{b\to\infty} {}_2F_1\left(a; b; c; \frac{x}{b}\right)$ = ...
8. Generating function for Legendre polynomial $P_n(x)$ is .....
9. The value of $Q_{n+1}' - Q_{n-1}'$ is .....
10. The value of $H_{2n}'(0)$ is .....