Roll No. ……………………..

MAT–502

Real Analysis and Topology

M. Sc. Mathematics (MSCMAT–12)

First Year, Examination, 2017

Time : 3 Hours **Max. Marks : 60**

Note : This paper is of **sixty (60)** marks containing **three (03)** sections A, B and C. Attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. If $\{A_1, A_2, A_3.........\}$ is a countable family of subsets of R, then prove that :

$$m^* \bigcup_{n=1}^{\infty} A_n \le \sum_{n=1}^{\infty} m^*(A_n).$$

2. Prove that if A is a closed and bounded set of real numbers, then every open covering of A has a finite subcovering.

3. Show that theorem of bounded convergence applies to

$$f_n(x) = \frac{nx}{1+n^2x^2},\ 0 \le x \le 1.$$

4. If the trigonometric series

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx)$$

converges uniformly on $[-\pi, \pi]$ to the integrable function f, then it is the Fourier series of f.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (05) marks each. Learners are required to answer *four* (04) questions only.

1. Let N be a set of all natural numbers. Prove that the relation R defined by setting aRb (a, b ∈N) iff a divides b is a partial order relation on N.
2. Prove that the union of enumerable collection of enumberable sets is also enumberable and hence deduce that $n.a = a$.
3. Let f and g be measurable function defined over a measurable set E. Show that $f + g$, $f - g$ are measurable function over E.
4. Let $\langle f_n \rangle$ be a sequence of measurable functions which converge in measure to f, then $\exists$ a subsequence $\langle f_{nk} \rangle$ of $\langle f_n \rangle$ which also converges to f almost everywhere.
5. Let f, $g \in L^p[a, b]$; $1 \leq p \leq \infty$, then prove that $(f + g) \in L^p[a, b]$.
6. Show that each second countable space is first countable but converse is not true.
7. Show that every sequentially compact topological space is countably compact.

8. Prove that a topological space (X, ζ) is Hausdorff if and only if every net in X can converge to at most one point.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

1. Card A .... card P(A), P (A) being power set of a set A.
2. A set A is Lebesgue measurable if and only it its complement A^c is measurable. (True/False)
3. Every closed and bounded interval on R is compact where R is the usual topology. (True/False)
4. A normed linear space is complete iff every summable sequence is summable. (True/False)
5. If *f* is bounded function defined on [*a, b*] and *f* is R-integrable, on [*a, b*] then *f* is also L-integrable on [*a, b*]. (True/False)
6. Every bounded measurable function defined on [*a, b*] is .... integrable over [*a, b*].
7. The cantor's set is totally disconnected. (True/False)
8. A countably compact topological space is ....
9. Close subset of compact sets are ....
10. A subset A of R is compact if and only if A is bounded and closed. (True/False)