Roll No. ……………………..

MAT–501

Advanced Algebra

M. Sc. MATHEMATICS (MSCMAT–12)

First Year, Examination, 2017

Time : 3 Hours **Max. Marks : 60**

Note : This paper is of **sixty (60)** marks containing **three (03)** sections A, B and C. Learners are required to attempt the questions contained in these sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of fifteen (15) marks each. Learners are required to answer *two* (02) questions only.

1. Let H and N be two subgroups of G, such that N is normal in G. Then $H \cap N$ is a normal subgroup of H and

$$\frac{H}{H \cap N} \cong \frac{HN}{N}$$

2. Define Euclidean ring. Let R be Euclidean ring, then every non-zero element in R is either a unit or can be written as the product of finite number of prime elements of R.

3. Define Field Extensions. If K is a finite field extension of a field F and L is a finite filed extension of K, then L is a finite field extension of F and

$$[L : F] = [L : K]\,[K : F]$$

4. Define 'direct sum' of vector subspace. Let V be a finite dimensional inner product space and W be its subspace. Then V is direct sum of W and $W^{\perp}$ symbolically $V = W \oplus W^{\perp}$.

Section–B

(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of five (05) marks each. Learners are required to answer *four* (04) questions only.

1. Consider two groups $(z_2, +_2)$ and $(z_3, +_3)$, where $z_2 = \{0, 1\}$ and $z_3 = \{0, 1, 2\}$. Then show that $z_2 \times z_3$ will be a group order 6.
2. Show that 'conjugacy on a group G in an equivalence relation'.
3. Define subnormal series. Show that symmetric group S_4 is solvable.
4. Define 'Modules'. Show that a ring R is an R-module over its subring.
5. If $B = e_1 = (0,1), e_2 = (0,1)$ is a usual basis R^2. Determine its dual basis.
6. If K is a finite extension of degree $n = [K : F]$ over F, then show that every element u of K has over F a degree which is a divisor of n.
7. Show that any algebraic extension of a finite field F is a separable extension.

8. A linear transformation $t : V \to V$ is invertible iff matrix of t relative to some bases B of V is invertible.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each. All the questions of this section are compulsory.

Fill in the blanks :

1. A homorphism of a group G to a group G' is called ...... if it is bijective.
2. Any two conjugate classes of a group are either disjoint or ....
3. The element $x^{-1}y^{-1}xy$ is called .... of x and y.
4. In a Euclidean ring R, 1 is an associate of ....
5. If $t : V \to V'$ be a linear transformation. If t is one-one, then ker (t) = ...

Write T for True and F for False :

6. An $n \times n$ square matrix A over a field F is invertible then rank (A) = n.
7. If $\lambda \neq 0$ is an eigen value of A, then $\frac{1}{\lambda}$ is an eigen value of A^T.
8. In Euclidean space R^2, angle θ between (5, 1) and (–2, 3) is $\frac{\pi}{2}$.
9. An orthogonal matrix is always singular.
10. An infinite abelian group does not have a composition series.