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Examination Session June-2022

(Fourth Semester)

MT-609

M.A./M.Sc. MATHEMATICS (MSCMT/MAMT)

[Integral Equations]

Time : 2 Hours] **[Max. Marks : 40**

Note : This paper is of Forty (40) marks divided into two (02) Section A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION—A

(Long-Answer-Type Questions)

Note : Section 'A' contains five (05) long-answer-type questions of Ten (10) marks each. Learners are required to answer any two (02) questions only. 2×10=20

1. (a) Show that the function $g(x) = \sin \frac{x}{2}$ is a solution of the Fredholm integral equation

$$g(x) \quad \frac{2}{4} \, {}_{0}^{x} K(x,t)g(t)dt \quad \frac{x}{2}$$

(b) Reduce the differential equation

$$\frac{d^2y}{dx^2} \quad 3\frac{dy}{dx} \quad 2y \quad 4\sin x$$

with the conditions $y(0) = 1, y'(0) = -2$ into a non-homogenous Volterra's integral equation of second kind.

2. Solve the integral equation

$$g(x) \quad f(x) \quad {}_{1}^{1}(xt \quad x^2t^2)g(t)dt$$

Also, find its resolvent kernel.

3. Solve by the method of successive approximation :

$$g(x) \quad \frac{3}{2}e^x \quad \frac{1}{2}xe^x \quad \frac{1}{2} \quad \frac{1}{2} \, {}_{0}^{1} t\, g(t)dt$$

6. Using the recurrence relations, find the resolvent kernels of $K(x, t) = \sin x \cos t$; $0 \leq x \leq 2\pi$; $0 \leq t \leq 2\pi$.

7. Explain the following :

(a) Symmetric Kernels

(b) Complex Hilbert space

(c) Orthonormal set

8. Prove that the eigenvalues of a symmetric kernel are real.

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4. Using Fredholm determinants, find the resolvent kernel, when

$$K(x,t) \quad xe^{t}, a \quad 0, b \quad 1$$

5. Using Hilbert schmidt theorem, find the solution of the symmertic integral equation :

$$g(x) \quad x^2 \quad 1 \quad \frac{3}{2} \int_{1}^{1} (xt \quad x^2t^2) g(t)\,dt$$

SECTION—B

(Short-Answer-Type Questions)

Note : Section 'B' contains eight (08) short-answer-type questions of Five (05) marks each. Learners are required to answer any four (04) questions only. 4×5 = 20

1. Show that the function $g(x) = xe^x$ is a solution of the Volterra integral equation :

$$g(x) \quad \sin x \quad 2 \int_{0}^{x} \cos(x \quad t) g(t)\,dt$$

4. Using Fredholm determinants, find the resolvent kernel, when

$$K(x,t) \quad xe^{t}, a \quad 0, b \quad 1$$

5. Using Hilbert schmidt theorem, find the solution of the symmertic integral equation :

$$g(x) \quad x^2 \quad 1 \quad \frac{3}{2} \int_{1}^{1} (xt \quad x^2t^2) g(t)\,dt$$

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2. Solve the homogeneous Fredholm integral equation of the second kind :

$$g(x) \qquad {}_{0}^{2} \ \sin(x \quad t) g(t) dt$$

3. Solve the equation

$$g(x) \quad 1 \qquad {}_{0}^{\overline{\ }\,2} \cos(x \quad t) g(t) dt$$

and find its eigen values.

4. Solve for $f(x)$ the integral equation

$$_{0} \ f(x) \cos px \ dx \qquad \begin{matrix} 1 \quad p, & 0 \quad p \quad 1 \\ 0, & p \quad 1 \end{matrix}$$

Hence deduce that :

$$_{0} \ \frac{\sin^2 t}{t^2} dt \qquad \frac{\ }{2}$$

5. Find the resolvent kernel of the Volterra integral equation with the kernel :

$$K(x,t) \qquad \frac{2 \quad \cos x}{2 \quad \cos t}$$

2. Solve the homogeneous Fredholm integral equation of the second kind :

$$g(x) \qquad {}_{0}^{2} \ \sin(x \quad t) g(t) dt$$

3. Solve the equation

$$g(x) \quad 1 \qquad {}_{0}^{\overline{\ }\,2} \cos(x \quad t) g(t) dt$$

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