# C197

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# MT-605

## Mathematical Programming-I

MA/M.Sc. Mathmatics (MAMT/MSCMT-20)

3rd Semester Examination, 2022 (June)

**Time : 2 Hours]** **Max. Marks : 40**

**Note :** This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

## SECTION–A

## (Long Answer Type Questions)

**Note :** Section 'A' contains Five (05) long answer type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

**1.** Using bounded variable technique, solve the following l.P.P

Max $z = x_1 + 3x_2$

s.t. $x_1 + x_2 + x_3 \leq 10$

$$x_1 - 2x_3 \geq 0$$

$$2x_2 - x_3 \leq 10$$

and $0 \leq x_1 \leq 8,\ 0 \leq x_2 \leq 4,\ x_3 \geq 0$

**2.** (a) Find the dimension of a rectangular parallelopiped with largest volume whose sides are parallel to the coordinate planets, to be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(b) Obtain the necessary conditions for the optimum solution of the following non-linear programming problem :

Min. $Z = f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$

Subject to the constraints: $x_1 + x_2 = 7$ and $x_1, x_2 > 0$

**3.** Solve the following non linear programming problem using the method of Lagrangian multipliers :

Minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

Subject to $4x_1 + x_1^2 + 2x_3 = 14$

$x_1, x_2, x_3 \geq 0$

**4.** (a) Define with examples

(i) Closed and Open set.

(ii) Convex set.

(b) Prove that a semi definite quadratic form $f(x) = X^T AX$ is a convex function over $\mathbb{R}^n$.

5. Find the optimum integer solution to the I.P.P

Max $z = x_1 + 2x_2$

s.t. $2x_2 \leq 7$

$x_1 + x_2 \leq 7$

$2x_1 \leq 11$

$x_1, x_2$ are integers and greater than equal to 0.

## SECTION–B

## (Short Answer Type Questions)

**Note :** Section 'B' contains Eight (08) short answer type questions of Five (05) marks each. Learners are required to answer any Four (04) questions only. (4×5=20)

1. Show that $f(x) = x^2$ is a convex function.

2. Solve the following linear programming problem by revised simplex method

Max $z = 2x_1 + x_2$

s. t. $3x_1 + 4x_2 \leq 6$

$6x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

3. Explain All I.P.P. algorithm or cutting plane algorithm.

**4.** Solve the following I.P.P by branch and bound technique

Max $z = x_1 + x_2$

s.t. $3x_1 + 2x_2 \leq 12$

$x_2 \leq 12$

$x_1, x_2 \leq 0$ and integers.

**5.** Determine the sign of definiteness for each of the following matrices.

(a) $\begin{bmatrix} 3 & 1 & 2 \\ 1 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 & 2 \\ 1 & -3 & 3 \\ 2 & 0 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 0 & 2 \\ 1 & -1 & 3 \\ 2 & 5 & -5 \end{bmatrix}$

6. Use method of Lagrangian multipliers to solve the following nonlinear programming problem:

Optimize $f(X) = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$
Subject to $x_1 + x_2 + x_3 = 20$

$x_1, x_2, x_3 \geq 0$

Does the solution maximize or minimize the objective function?

7. Obtain the necessary and sufficient conditions for the optimum solution of the following NLPP

Minimize $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

Subject to $x_1 + x_2 + x_3 = 15,\ 2x_1 - x_2 + 2x_3 = 30$

$x_1, x_2, 2x_3 \geq 0$

8. Prove that :

(a) A hyperplane is a convex set

(b) The closed half spaces $H_1 = \{X : CX \geq Z\}$ and $H_2 = [X: CX \leq Z)$ are convex sets.