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MT-604

Integral Transforms

MA/M.Sc. Mathematics (MAMT/MSCMT-20)

3rd Semester Examination, 2022 (June)

Time : 2 Hours] **Max. Marks : 40**

Note : This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. If $f(t)$ is a periodic function with period $T > 0$ i.e., $f(u + T) = f(u)$, $f(u + 2T) = f(u)$, etc. then prove that

$$L[f(t); p] = \frac{1}{1-e^{-pt}} \int_0^T e^{-pt} f(t)\, dt.$$

2. Define Mellin transform. If $M\{f(x); p] - F(p)$, then find the following :

(a) $M\{f(ax); p\}$.

(b) $M\{x^a f(x); p\}$.

(c) $M\{f(x^a); p\}$.

(d) $M\{\log x\, f(x); p\}$.

3. If $f(x) = \frac{e^{-ax}}{x}$ then find,

(a) The Hankel transform of order zero of the function

$$\frac{d^2 f}{dx^2} + \frac{1}{x}\frac{df}{dx}.$$

(b) The Hankel transform of order one of $\frac{df}{dx}$.

4. Use partial fractions to find the inverse Laplace transform of $\frac{p^2}{p^4 + 4a^4}$.

5. Find $f(t)$, if its Fourier sine transform is $\frac{p}{(1+p^2)}$.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Five (05) marks each. Learners are required to answer any Four (04) questions only. (4×5=20)

1. Find the Laplace transform of the function $f(t) = t^2 e^t \sin 4t$.

2. Apply convolution theorem to prove that

$$B(m, n) = \int_0^1 u^{m-1} (1-u)^{n-1}\, du = \frac{\Gamma m\, \Gamma n}{\Gamma (m+n)}, \ (m > 0, n > 0)$$

Hence deduce that

$$\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2} B(m, n) = \frac{\Gamma m\, \Gamma n}{2\Gamma (m+n)}$$

where $B(m, n)$ is called Beta function.

3. If $\bar{f}(p)$ is the Laplace transform of $f(t)$ i.e., $L[f(T); p] = \bar{f}(p)$ then prove that $L[t^n f(t); p] = (-1)^n \frac{d^n}{dp^n} \bar{f}(p)$.

4. Find the relation between Fourier transform and Laplace transform.

5. If $F(p)$ is the Mellin transform of $f(x)$ i.e., $M\{f(x); p\} = F(p)$

then show that, $M\left\{\int_0^x f(u)\,du;\, p\right\} = -\frac{1}{p}F(p+1)$.

6. Find the Hankel transform of following

(a) $\frac{\cos ax}{x}$.

(b) $\frac{\sin ax}{x}$.

taking $xJ_0(p_x)$ as the kernel.

7. Solve the differential equation using Laplace transform

$(D^2 + 9)y = \cos 2t$; if $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = -1$.

8. Solve $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$ subject to the condition, $u(0, t) = 0$, $u(2, t) = 0$, $u(x, 0) = 20 \sin 2\pi x - 10 \sin 5\pi x$ and $u_t(x, 0) = 0$