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MT-601

Analysis and Advanced Calculus-I

MA/M.Sc. Mathematics (MAMT/MSCMT-20)

3rd Semester Examination, 2022 (June)

Time : 2 Hours] **Max. Marks : 40**

Note : This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. If T be a linear transformation from a normed linear space N into the normed space N', Then prove that the following statement are equivalent:

(a) T is continuous.

(b) T is continuous at the origin i.e. $x_n \to 0 \Rightarrow T(x_n) \to 0$.

(c) T is bounded i.e., $\exists$ real $K \geq 0$ s. t. $\|T(x)\| \leq K\|x\|$ for all $x \in N$.

(d) If $S = \{x : \| x \| \leq 1\}$ is the closed unit sphere in N, then its image T(S) is bounded set in N′.

2. Prove that if B and B′ are Banach spaces and T is a linear transformation of B into B′, then T is continuous iff its graph is closed.

3. Prove that if M be linear subspace of a normed linear space N and f is a functional defined on M, then f can be extended to a functional f_0 defined on the whole space N s.t. $\|f_0\| = \|f\|$.

4. If H be a Hilbert space and $\{e_i\}$ be an orthonormal set in H, then prove that the following statement are equivalent

(a) $\{e_i\}$ is complete.

(b) $x \perp \{e_i\} \Rightarrow x = 0$.

(c) If x is an arbitrary vector in H, then $x = \Sigma(x, e_i)e_i$.

(d) If x is an arbitrary vector in H, then $\| x \|^2 = \Sigma\, |(x, e_i)|^2$.

5. (a) Define with example

(i) Inner product space.

(ii) Hilbert Space.

(b) Give an example of inner product space which is not Hilbert space.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Five (05) marks each. Learners are required to answer any Four (04) questions only. (4×5=20)

1. Prove that Every normed linear space is a metric space.

2. Let N and N′ be normed linear spaces over the same scalar field and let T be a linear transformation of N into N′. Then prove that T is bounded if it is continuous.

3. Prove that If B and B′ are Banach spaces. If T is a continuous linear transformation of B onto B′, then T is an open mapping.

4. Prove that If M be a closed linear subspace of a normed linear space N and x_0 be a vector in N, but not in M with the property that the distance from x_0 to M i.e. $d(x_0, M) = d > 0$, then there exists a bounded linear functional $F \in N^*$ s. t. $\| F \| = 1$, $F(x_0) = d$ and $F(x) = 0 \; \forall \; x \in M$ i.e. $F(M) = \{0\}$.

5. Define with example :

(a) Normed linear Space.

(b) Compactness.

(c) Banach Space.

6. If x and y are any two vectors in an inner product space X, then prove that $| < x, y > | \leq \| x \| \, \| y \|$.

7. If M is a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$.

8. State and Prove Pythagorean theorem.
