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MT-510

Mechanics-II

MA/M.Sc. Mathematics (MAMT/MSCMT-20)

2nd Semester Examination, 2022 (June)

Time : 2 Hours] **Max. Marks : 40**

Note : This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. State and prove the principle of least action for a conservation holonomic system.

2. Show that for an incompressible fluid, the equation of continuity becomes $\operatorname{div}\vec{q} = 0$.

3. Derive the equation of continuity in cartesian coordinate system.

4. A mass of fluid is in motion so that the lines of motion on the surface of coaxial cylinders, show that the equation of continuity is $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (\rho u)}{\partial \theta} + \frac{\partial (\rho v)}{\partial z} = 0$ where u, v are the velocity perpendicular and parallel to z.

5. A circular disc, of radius a, has a thin rod pushed through its center perpendicular to its plane, the length of the rod being equal to the radius of the disc. Show that the system cannot spin with the rod vertical unless the angular velocity (n) is greater than $\sqrt{\frac{20g}{a}}$.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Five (05) marks each. Learners are required to answer any Four (04) questions only. (4×5=20)

1. For a two-dimensional flow the velocities at a point in the fluid may be expressed in the Eulerian coordinates by $u = x + y + 2t$ and $v = 2y + t$.

   Determine the Lagrange coordinate as function of the initial position x_0 and y_0 and the time t.

2. Find the stream lines and path lines of the particles of the velocity field.

$$u = \frac{x}{1+t}, \ v - y \text{ and } w = 0.$$

3. A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis.

   Show that the equation of continuity is $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial \theta} = 0.$

   Where w be the angular velocity of a particle whose azimuthal angle is θ at time t.

4. State and proof the Bernoulli's theorem.

5. If u, v, w are the velocity components in the direction of three axes at any point (x, y, z). Derive the differential equation of stream lines in cartesian coordinates.

6. Define stream function. Prove that the stream function is constant along a stream line.

7. Define the following characteristics of fluid

   (a) Density.

   (b) Pressure.

   (c) Compressibility.

   (d) Viscosity.

8. Find the equation of stream lines passing through the point (1, 1, 1) for an incompressible flow $\vec{q} = 2x\hat{i} - y\hat{j} - z\hat{k}$.