C191

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MT-509

Differential Geometry and Tensor-II

MA/M.Sc. Mathematics (MAMT/MSCMT-20)

2nd Year Examination, 2022 (June)

Time : 2 Hours] **Max. Marks : 40**

Note : This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. Prove that the Christoffel symbols are not tensor quantities.

2. If A_{ij} is the *curl* of a covariant vector, prove that $A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$.

Show further that this expression is equivalent to

$$\frac{\partial A_{ij}}{\partial x^k} + \frac{\partial A_{jk}}{\partial x^i} + \frac{\partial A_{ki}}{\partial x^j} = 0. \text{ If } A_{ij} = B_{i,j} - B_{j,i}.$$

Prove that $A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$.

3. Show that the great circle on sphere are geodesic.

OR

Show that on the surface of a sphere, all great circles are geodesies while no other circle is a geodesic.

4. What is the Einstein Tensor? Prove that divergence of Einstein Tensor vanishes.

5. Define geodesic curvature. Find the geodesic curvature of the curve u = constant, on the surface

$$x = u\cos\theta, \quad y = u\sin\theta, \quad z = \frac{1}{2}au^2$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Five (05) marks each. Learners are required to answer any Four (04) questions only. (4×5=20)

1. On the general surface $\vec{r} = \vec{r}(u, v)$ what is the necessary and sufficient condition that the curve $v = c$(constant) be a geodesic?

2. Drive the formula for geodesic curvature of the form $K_g = \left[\hat{N}\,\vec{r}'\,\vec{r}''\right]$.

3. Prove that a symmetric tensor of second order has at most $\frac{N(N+1)}{2}$ different component in V_N.

4. If a metric of a V_3 is given by $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6(dx^1)(dx^2) + 4(dx^2)(dx^3)$

Find

(a) $|g_{ij}|$

(b) g^{ij}.

5. Prove that $\dfrac{\partial g^{mk}}{\partial x^l} = -g^{mj}\begin{Bmatrix} k \\ jl \end{Bmatrix} - g^{ki}\begin{Bmatrix} m \\ il \end{Bmatrix}.$

6. Prove that $k_g^2 + k_n^2 = k^2$.

7. Calculate the Christoffel symbols corresponding to the metric $dS^2 = (dx^1)^2 + G(x^1, x^2)(dx^2)^2$ where G is a function of x^1 and x^2.

8. Show that

(a) $g^{ij} g^{kl} dg_{ik} = -dg^{jl}$.

(b) $g_{ij} g_{kl} dg^{ik} = -dg_{jl}$.
