C190

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MT-508

Special Functions

MA/M.Sc. Mathematics (MAMT/MSCMT-20)

2nd Semester Examination, 2022 (June)

Time : 2 Hours] **Max. Marks : 40**

Note : This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. Solve the Legendre's equation

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)\,y=0$$

2. Show that $P_n(x) = \frac{1}{2^n n! dx^n}(x^2 - 1)^n$.

3. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

4. Show that $\frac{e^{-\frac{xt}{1-t}}}{1-t} = \sum_{n=0}^{\infty} L_n(x).t^n$.

5. Solve the differential equation in series $(1 - x^2)y_2 - xy_1 + 4y = 0$.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Five (05) marks each. Learners are required to answer any Four (04) questions only. (4×5=20)

1. Prove that ${}_2F_1\left[\frac{a}{2}, \frac{a}{2} + \frac{1}{2}; \frac{1}{2}; z^2\right] = \frac{1}{2}$ $[(1 - z)^{-a} + (1 - z)^{-a}]$.

2. Prove that $(2n + 1)P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)n$.

3. Prove that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$.

4. Show that $xJ'_n(x) = nJ_n(x) - xJ_{n+1}(x)$.

5. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}.\sin x$.

6. Prove that $e^{2xt - t^2} = \sum_{n=0}^{\infty} \dfrac{t^n}{n!} H_n(x)$ valid for all finite x and t.

7. Show that $H'_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.

8. Find the value of

(a) $\displaystyle\int_0^\infty e^{-x} L_3(x) L_5(x) dx$.

(b) $\displaystyle\int_0^\infty e^{-x} [L_4(x)]^2 dx$.