

# C189

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## MT-507

### Topology

MA/M.Sc. Mathematics (MAMT/MSCMT-20)

2nd Semester Examination, 2022 (June)

**Time : 2 Hours]**

**Max. Marks : 40**

**Note :** This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

### SECTION–A

#### (Long Answer Type Questions)

**Note :** Section 'A' contains Five (05) long answer type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. Prove that a second countable space is always first countable space, but converse is not true.

2. Prove that the homeomorphism is an equivalence relation in the family of topological spaces.
3. State and prove Heine-Borel Theorem.
4. Show that the property of a space being a Hausdorff space is a hereditary property.
5. Prove that a topological space is Hausdorff iff every net in the space converge to at most one point.

## SECTION-B

### (Short Answer Type Questions)

**Note :** Section 'B' contains Eight (08) short answer type questions of Five (05) marks each. Learners are required to answer any Four (04) questions only. (4×5=20)

1. Define Topological space and give an example of Discrete topology and Indiscrete topology (Trivial topology).
2. Let  $X = \{1,2,3,4\}$  and  $A = \{\{1,2\}, \{2,4\}, \{3\}\}$ . Determine the topology on  $X$  generated by the elements of  $A$  and hence determine the base for this topology.
3. Let  $X = \{0,1,2\}$ ,  $\tau = \{\emptyset, X, \{0\}, \{0,1\}\}$ . Let  $f$  be a continuous map of  $X$  in to itself such that  $f(1) = 0, f(2) = 1$ , what is  $f(0) = ?$ .

4. Prove that if every two points of a subset  $A$  of a topological space  $X$  are contained in some connected subset of  $A$ , then  $A$  is connected.
  5. Give an example to show that the quotient space of a Hausdorff space need not be a Hausdorff.
  6. Define Net, Ultranet, subnet and Filter with example.
  7. Let  $X = \{1,2,3,4\}$  and  $C = \{\{1,2\}, \{1,3\}\}$ , then find base and filter taking  $C$  as a subbase.
  8. Let  $X = \{1,2,3\}$ ,  $\tau = \{\phi, \{0\}, X\}$  and  $Y = \{a, b, c\}$ ,  $V = \{\emptyset, \{a\}, \{a, c\}, Y\}$ . Find a base for the product topology  $W$  on  $X \times Y$ .
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