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MT-504

Differential Geometry and Tensor-I

M.A/M.Sc. Mathematics (MAMT/MSCMT-20)

Ist Semester, Examination, June 2022

Time : 2 Hours Max. Marks : 40

Note: This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION – A

(Long-answer – type questions)

Note : Section 'A' contains Five (05) long-answer-type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. (a) Prove that the Oscillating plane at $(x_1, y_1\ z_1)$ on the curve of intersection of the cylinders $x^2 + z^2 = a^2$, $y^2 + z^2 = b^2$ is given by

   x P.T.O.

   (b) For the curve $x = 3t$, $y = 3\ t^2$, $z = 2t^3$.

Show that $\rho = -\sigma = 3/2\ (1 + 2t^2)^2$

2. (a) Prove that

$$x^{111^2} + y^{111^2} + z^{111^2} = \frac{1}{\rho^2\sigma^2} + \frac{1+\rho^{12}}{\rho^4}$$

Where dashes denote differentiation with respect to 'S'.

(b) Find the principal radii at the origin of the surface

$$2z = 5x^2 + 4xy + 2y^2$$

3. (a) If the curve lies on a sphere, show that ρ and σ are connected by

$$\frac{\rho}{\sigma} + \frac{d}{ds}(\rho^1\sigma) = 0$$

(b) Prove that for the curve $x = r\cos\theta$, $y = r\sin\theta$, $z = 0$, $ds^2 = dr^2 + r^2\,d\theta^2$.

P.T.O.

4. (a) Examine whether the parametric cures

$x = b \sin u \cos v, \quad y = b \sin u \sin v,$

$z = b \cos u$ on a sphere of radius b

constitute an orthogonal system.

(b) Show that

(i) $S^i_j \; S^j_k = S^i_k$

(ii) $\frac{\partial x^k}{\partial \bar{x}^i} , \frac{\partial \bar{x}^i}{\partial x^j} = S^k_j$

5. (a) Define the following

(i) Principal sections of surface

(ii) Principal Radius of curvature.

(b) State and Prove existence and uniqueness theorem.

SECTION – B

(Short – answer – type questions)

Note: Section 'B' contains Eight (08) short – answer type questions of Five (5) marks each. Learners are required to answer any Four (04) questions only.

(4×5 = 20)

P.T.O.

1. Prove that

$$k = \frac{|\overset{-1}{r} \times \overset{-11}{r}|}{|\overset{-1}{r}|^{3}}$$

2. Find and classify the singular points of the surface

$$xyz - a^2 (x+y+z) + 2a^3 = 0$$

3. Show that the surface $e^z \cos x = \cos y$ is minimal surface.

4. Find the asymptotic lines on the surface

$$z = y \sin x$$

5. Show that the tangents at any point of the curve whose equation are

$$x=3t \quad y=3t^2 \quad z=2t^3$$

6. By considering the value of $rt - s^2$, determine if the surface is developable.

P.T.O.

7. Define minimal surface. Find the condition that the surface of revolution given by $x = u \cos\theta$, $y = u \sin\theta$, $z = f(u)$ be minimal surface.

8. Solve $t - qx = x^2$
