Total Pages: 4 Roll No. -----------------

# MT-503

# Differential Equation and Calculus of Variation

M.A/M/Sc. Mathematics (MAMT/MSCMT-20)

IstSemester, Examination, June 2022

Time : 2 Hours Max. Marks :40

Note : This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

## SECTION – A

## (Long-answer – type questions)

Note: Section 'A' contains Five (05) long-answer-type questions of ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. Solve : $3r + 4s + t + (rt - s^2) = 1$.

P.T.O.

2. Reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form and hence solve it.

3. Find the eigenvalues and eigenfunctions for the following boundary value problem:

$y'' - 4y' + (4 - 9\mu)\, y = 0,\ y(0) = 0,\ y(a) = 0,$

Where 'a' is a positive real constant.

4. Extremize : $I[y(x)] = \int_1^e (xe^y - ye^x)dx,\ y(1) = 1,$

$y(e) = e.$

5. Obtain the surface of minimum area, stretched over a given closed curve C, enclosing the domain D in the xy plane.

## SECTION – B

## (Short – answer – type questions)

Note: Section 'B' contains Eight (08) short – answer type questions of five(05) marks each. Learners are required to answer any Four (04) questions only. $(4\times5 = 20)$

P.T.O.

1. Find the general solution of the Riccati's equation

$$\frac{dy}{dx} = 2 - 2y + y^2$$

Whose one particular solution is (1+tan x).

2. Solve: $(y^2 + z^2 - x^2)dx - 2xydy - 2xzdz = 0$

3. Solve:

$$(2xz - yz)dx + (2yz - xz)dy - (x^2 - xy + y^2)dz = 0$$

4. Solve: $2yq + y^2t = 1$

5. Find the characteristics of

$x^2r + 2xys + y^2t = 0$

6. Test for extremum of the functional

$F[y(x)] = \int_a^b [\cos y - xy' \sin y]dx$

P.T.O.

7. Find the extremals of the functional

$$I[y, z] = \int_0^{\pi/2} [y^2 + z^2 + 2yz]dt.$$

With the boundary condition y(0)=0, y($\pi$/2)=-1;

z(0)=0, z($\pi$/2)=1.

8. Prove that the extremal of $\int_a^b y(1+y'^2)^{1/2}\,dx$ is the catenary y=acosh(ax+b).

----------------------------