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MT-502

Real Analysis

M.A/M.Sc. Mathematics (MAMT/MSCMT-20)

I[st]Semester, Examination, June 2022

Time : 2 Hours Max. Marks : 40

Note : This paper is of Forty (40) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION – A

(Long-answer – type questions)

Note : Section 'A' contains Five (05) long-answer-type questions of Ten (10) marks each. Learners are required to answer any Two (02) questions only.

(2×10=20)

1. Prove that the union of two measurable sets is a measurable set.

P.T.O.

2. Let $\langle E_i \rangle$ be an infinite decreasing sequence of measurable sets, that is $E_1 \supset E_2 \supset E_3 \ldots$, Let

$m(E_i) < \infty$ for at least one $i \in N$. Then prove that

$$m\left(\bigcap_{i=l}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n).$$

3. Define measurable function. Let f and g be measurable functions defined on a measurable set E and c be a constant. Then prove that the functions $f \pm g$, cf and $f.\ g$ are measurable.

4. Prove that the necessary and sufficient condition for a bounded function f defined on the interval [a, b], to be L-integrable over [a, b] is that given $\in >0$, there exists a measurable partition P of [a, b] such that $U(f, P) - L(f, P) < \in$.

5. Let $1 \leq p \leq \infty$ and q be a non-negative real number such that $\frac{1}{p} + \frac{1}{q} = 1$. if $f \in L^p$ and $g \in L^q$, then show that

   (i) $f.$ g is summable

   (ii) $$\int_E |f(x)g(x)|\,dx \leq \left[\int_E |f(x)|^p\,dx\right]^{1/p} . \left[\int_E |g(x)|^q\,dx\right]^{1/q}$$

P.T.O.

SECTION – B

(Short – answer – type questions)

Note : Section 'B' contains Eight (08) short – answer type questions of Five(5) marks each. Learners are required to answer any Four (04) questions only.

(4×5 = 20)

1. Prove that the outer measure is a translation invariant.

2. If E_1 and E_2 are two measurable sets, then prove that $m^*(E_1 \cup E_2) + m^*(E_1 \cap E_2) = m^*(E_1) + m^*(E_2)$

3. Show that a function f on a set E is measurable if and only if for any rational number $r \in Q$, the set $\{x \in E : f(x) < r\}$ is measurable

4. Let $\langle f_n \rangle$ be a sequence of a measurable functions defined on the measurable set E, then prove that $\limsup_n \langle f_n \rangle$ and $\liminf_n \langle f_n \rangle$ are also measurable on E.

P.T.O.

5. Let f be a bounded function defined on a measurable set E. If P and P' are two measurable partitions of E such that P' is a refinement of P, then prove that

(i) $L(f, P) \leq L(f, p)$

(ii) $U(f, P) \geq U(f, P)$

6. Let f and g be bounded measurable functions on a measurable set E and $f = g$ almost everywhere on E. Then show that $\int_E f(x)dx = \int_E g(x)dx$.

7. Prove that a measurable function f is summable on E if and only if $|f|$ is summable and also in this case $|\int_E f(x)dx| \leq \int_E |f(x)dx$.

8. Show that the L^p space is a metric space.
