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MT-501

Advanced Algebra-1

M.A./M.Sc. Mathematics (MAMT/MSCMT-20)

1st Semester Examination June 2022

Time : 2 Hours Max. Marks : 40

Note : This Paper is of forty (40) marks divided into two (02) Section A and B. Attempt the questions contained in these sections according to the detailed instructions given there in.

Section-A

(Long Answer-type questions)

Note: Section 'A' contains Five (05) Long-answer type questions of ten (10) marks each. Learners are required to answer any two (02) questions only.

(2 x 10 = 20)

Q.1. Let G be a group. If H and K are two subgroup of G such that H and K are normal to G and $H \cap K = \{e\}$, then prove that

(i) HK is the internal direct product of H and K.

(ii) $HK \cong H \times K$.

P.T.O.

Q.2. Let G be a finite group, then show that $o(G)=\sum\frac{o(G)}{o(N(a))}$, where summation runs over one element **a** in each conjugate class.

Q.3. Define refinement of a subnormal series. Prove that any two subnormal series for the group G have equivalent refinements.

Q.4. Define Euclidean ring. Prove that the ring of Gaussian integers is a Euclidean ring.

Q.5. Let V be n-dimensional vector space over a field F and $B=\{b_1,b_2,.......b_n\}$ be a basis for V, then prove that for any n scalars $\lambda_1\lambda_2.......,\lambda_n\in F$ there exists a unique linear functional $F\in V^*$ such that $f(b_i)=\lambda_i$.

Section-B

(Short Answer-type questions)

Note: Section 'B' contains Eight (08) Short-answer type questions of five (05) marks each. Learners are required to answer any four (04) questions.

(4 x 5 = 20)

Q.1. Prove that conjugacy on a group G is an equivalence relation.

P.T.O.

Q.2. Define derived subgroup of a group G. Let $G^{(1)}$ be the first derived subgroup of the group G, then prove that $G^{(1)}$ is normal to G and quotient group $G/G^{(1)}$ is abelian.

Q.3. Define composition series. Show that every finite group has a composition series.

Q.4. State and prove Jordan-Holder theorem.

Q.5. Prove that every Euclidean ring is a Principal Ideal Domain.

Q.6. Define R-module. Prove that a ring R is an R-module over its subring.

Q.7. If $B = \{b_1 = (-1,1,1), b_1 = (1,-1,1), b_1 = (1,1,-1)\}$ is a basis of $V_3(R)$, then find the basis dual to B.

Q.8. Let $F \subset K \subset L$ be three fields. If L is an algebraic extension of K and if K is an algebraic extension of F. then prove that L is an algebraic extension of F.
