C156

MSCPH-501

Mathematical Physics

M.Sc. PHYSICS(MSCPH-21)

Ist Semester Examination, 2022 (June)

Time : 2 Hours]   **Max. Marks : 80**

Note : This paper is of Eighty (80) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Twenty (20) marks each. Learners are required to answer any Two (02) questions only.

$(2×20=40)$

1. What are curvilinear coordinates? Calculate (i) gradient (ii) divergence (iii) curl, for cylindrical coordinates system.

2. Calculate the value of integral $\int_0^{2\pi} \dfrac{\cos 2\theta}{5 + 4\cos\theta}\, d\theta$.

3. What are Christofell's 3-index symbols? Establish relation between Christofell's symbols of first and second kind.

4. Find the Solution of $\dfrac{d^2 y}{dx^2} - 2\dfrac{dy}{dx} + y = \dfrac{e^x}{x^2 + 1}$.

5. Find series solution of Legendre differential equation :

$$(1 - x^2)\, y'' - 2xy' + n(n + 1)y = 0.$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Ten (10) marks each. Learners are required to answer any Four (04) questions only. (4×10=40)

1. (a) Show that the matrix $\dfrac{1}{6}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$ is idempotent.

(b) Show that if A is idempotent, then

$$(1 + A)^n = 1 + (2^n - 1)A.$$

2. Find Taylor Expansion of $f(z) = \dfrac{2z^3 + 1}{z^2 + z}$ about the point $z = i$.

3. Show that the sum or difference of two tensors of the same rank and same type is also a tensor of the same rank and same type?

4. What do you understand by Contraction of tensors? Discuss it with an example.

5. Show that $(n + 1) P_{n+i}(x) = (2n + 1)xP_n(x) - n P_{n-1}(x)$.

6. Show that

$$\lim_{z \to 0} \frac{d^2}{dz^2} \left[(1 - z)^{-1} \exp\left(-\frac{x}{1 - z} \right) \right] = (x^2 - 4x + 2)e^{-x}.$$

7. Find the Laplace transform of $\sin^3 2t$.

8. Using Gauss divergence theorem find out $\iint\limits_{s} \vec{A} \cdot d\vec{s}$ where,

$A = x^3 i + y^3 j + z^3 k$ and s is a surface of a sphere defined by $x^2 + y^2 + z^2 = a^2$.

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