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MAMT-09

Integral Transforms and Integral Equations

MA/M.Sc. Mathematics (MAMT/MSCMT)

2nd Year Examination, 2022 (June)

Time : 2 Hours] **Max. Marks : 80**

Note : This paper is of Eighty (80) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Twenty (20) marks each. Learners are required to answer any Two (02) questions only.

(2×20=40)

1. Find fourier transform of $f(t)$ defined by $f(t) = \begin{cases} 1, & |t| < a \\ 0, & |t| > a \end{cases}$

and hence prove that $\int_0^\infty \frac{\sin^2 at}{t^2} dt = \frac{\pi a}{2}$.

2. State and prove Parseval's Theorem for Hankel transform.

3. Find the eigenvalues and eigenfunction of the homogeneous integral equation

$$g(x) = \lambda \int_0^{\pi} [\cos^2 x \cos 2t + \cos 3x \cos^3 t]\, g(t)dt$$

4. State and prove Hilbert-Schmidt Theorem.

5. Prove that

(a) If $L^{-1}\left\{\dfrac{p^2-1}{(p^2+1)^2};t\right\}$ t cost, then

find $L^{-1}\left\{\dfrac{9p^2-1}{(9p^2+1)^2};t\right\}$.

(b) $L^{-1}\left\{\dfrac{e^{-\frac{1}{p}}}{\sqrt{p}};t\right\} = \dfrac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ and hence deduce the value

of $L^{-1}\left\{\dfrac{e^{-\frac{a}{p}}}{\sqrt{p}};t\right\}$, where $a > 0$.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Ten (10) marks each. Learners are required to answer any Four (04) questions only. (4×10=40)

1. Explain

(a) Dirichlet's conditions.

(b) Convolution product of two functions.

(c) Kernel of the Mellin Transform.

(d) Hankel transform.

(e) Singular Integral equation.

2. Find the Laplace transform of :

(a) $\cosh^2 4t$.

(b) $(1 + te^{-t})^3$.

(c) $t^2 e^t \sin 4t$.

3. By means of resolvent kernel, find the solution of

$$g(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2}\, g(t)dt.$$

4. Solve $\dfrac{d^4y}{dx^4} - y = 1$, subject to conditions $y(0) = y'(0) = y''(0) = y'''(0) = 0$ using Laplace transform.

5. Prove that $M\{(1 + x^a)^{-b}; p\} = \dfrac{r\left(\frac{p}{a}\right) r\left(b - \frac{p}{a}\right)}{arb}$; $0 < \text{Re}(p) < \text{Re}(ab)$.

6. Solve $g(x) = e^x + \lambda \int_0^1 2e^x e^t g(t)dt$.

7. Find the resolvent kernels of $K(x, t) = (1 + x)(1 - t)$, $a = -1$, $b = 0$.

8. For the integral equation $g(x) = f(x) + \lambda \int_a^b K(x, t)\, g(t)dt$.

Find $D(\lambda)$ and $D(x, t: \lambda)$ for the kernel $k(x, t) = \sin x$; $a = 0$, $b = \pi$.