# C178

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# MAMT-06

**Analysis and Advanced Calculus**

MA/M.Sc. Mathematics (MAMT/MSCMT-19)

2nd Year Examination, 2022 (June)

**Time : 2 Hours]** **Max. Marks : 80**

**Note :** This paper is of Eighty (80) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

## SECTION–A
## (Long Answer Type Questions)

**Note :** Section 'A' contains Five (05) long answer type questions of Twenty (20) marks each. Learners are required to answer any Two (02) questions only.

(2×20=40)

**1.** If T be a linear transformation from a normed linear space N into the normed space N, prove that the following statement are equivalent :

(a) T is continuous.

(b) T is continuous at the origin i.e. $x_n \to 0 \Rightarrow T(x_n) \to 0$.

(c) T is bounded i.e., $\exists$ real $K \geq 0$ s.t. $\|T(X)\| \leq K\|x\|$ for all $x \in N$.

**2.** State and prove the Minkowski's Inequality.

**3.** If $\{e_1, e_2, ......., e_n\}$ be finite orthonormal set in a Hilbert space H, and $x$ be any vector in H, then prove that

(a) $\sum_{i=1}^{n} \left|(x, e_j)\right|^2 \leq |x|^2$ and

(b) $x - \sum_{i=1}^{n} (x, e_i)e_i \perp e_j \ \forall j$

**4.** Let X be a Banach space over the field K of scalars and let $f : [a, b] \to X$ and $g : [a, b] \to R$ be continuous and differentiable functions such that $\| Df(t) \| \leq Dg(t)$ at each point $t \in (a, b)$. Then prove that $\| f(b) - f(a) \| \leq g(b) - g(a)$.

**5.** State and prove Hahn-Banach theorem.

## SECTION–B

## (Short Answer Type Questions)

**Note :** Section 'B' contains Eight (08) short answer type questions of Ten (10) marks each. Learners are required to answer any Four (04) questions only. (4×10=40)

**1.** Prove that every normed linear space is a metric space.

2. Prove that every compact subset of a normed linear space is complete.

3. Let N and N′ be normed linear spaces and D be a subspace of N. Prove that a linear transformation $T : D \rightarrow N'$ is closed iff its graph $T_G$ is closed.

4. If $x$ and $y$ are any two vectors in an inner product space X, then prove that $|(x, y)| \leq \| x \| \| y \|$.

5. The inner product in a Hilbert space is jointly continuous i.e., if $x_n \rightarrow x$ and $y_n \rightarrow y$, then prove that $(x_n, y_n) \rightarrow (x, y)$ as $n \rightarrow \infty$.

6. Prove that if S is a non-empty subset of a Hilbert space H, then $S^{\perp}$ is a closed linear subspace of H and hence a Hilbert space.

7. If T is an operator on a Hilbert space H, then prove that the following conditions are equivalent :

(a) $T^*T = I$.

(b) $(Tx, Ty) = (x, y) \ \forall \ x, y \in H$.

(c) $\| Tx \| = \| x \| \ \forall \ x \in H$.

8. Prove that every convergent sequence in a normed linear space is a Cauchy sequence.