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MAMT-02

Real Analysis and Topology

M.Sc./M.A. Mathematics (MSCMT/MAMT-19)

Ist Year Examination, 2022 (June)

Time : 2 Hours] **Max. Marks : 80**

Note : This paper is of Eighty (80) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Twenty (20) marks each. Learners are required to answer any Two (02) questions only.

(2×20=40)

1. Prove that the outer measure of an interval is its length.

2. Prove that the necessary and sufficient condition for a bounded function f defined on an interval $[a, b]$ to be

L - integrable over $[a, b]$ is that given $\in > 0$,there exists a partition P of $[a, b]$ such that $U(f, P) - L(f, P) < \in$.

3. Let U be a collection of all subsets $G \subset R$, having the property that to each $x \varepsilon \mathbf{R} \exists \delta > 0$ such that open interval $(x - \delta, x + \delta) \subset G \ \forall \ x \varepsilon G$. Prove that U is a topology on **R**.

4. Prove that a second countable space is always a first countable space, but the converse is not true.

5. Let (X, τ) be a topological space and let A, B be non empty subsets of X then prove that

(a) $\varnothing' = \varnothing$.

(b) $x \in A' \Rightarrow x \in (A - \{x\})'$.

(c) $A \subset B \Rightarrow A' \subset B'$.

(d) $(A \cup B)' = A' \cup B'$.

(e) $(A \cap B)' \subset (A' \cap B')$.

where A' means derived set of A.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Ten (10) marks each. Learners are required to answer any Four (04) questions only. (4×10=40)

1. If E is a countable set, the prove that $m^*(E) = 0$.

2. If f is a bounded function defined on a measurable set E, and $m(\text{E}) = 0$. Then prove that $\int_{\text{E}} f(x)dx = 0$.

3. Show that the L^{P} space is a metric space .

4. Give one example of a topology on $\text{X} = \{a, b, c\}$ in which every open set is also a Closed set.

5. Define base for a topology.

6. Define Compact Topological Spaces.

7. For any two sets A and B if $m^*(\text{A}) = 0$, then prove that $\text{m}^*(\text{A} \cup \text{B}) = m^*(\text{B})$.

8. Define the following :

(a) Discrete Topology.

(b) Indiscrete Topology.

(c) Toplogical Space.
