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MAMT-01

Advanced Algebra

M.Sc./M.A. Mathematics (MSCMT/MAMT-19)

Ist Year Examination, 2022 (June)

Time : 2 Hours] **Max. Marks : 80**

Note : This paper is of Eighty (80) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Twenty (20) marks each. Learners are required to answer any Two (02) questions only.

(2×20=40)

1. Apply the Gram-Schmidt process to the vectors $e_1 = (1,0,1)$, $e_2 = (1,0,-1)$, $e_3 = (0,3,4)$, to obtain an orthonormal basis for $R^3(R)$ with the standard inner product.

2. Let T : $R^3 \to R^3$ be a linear transformation such that $T(a, b, c) = (3a + c, \ -2a + b, -a + 2b + 4c)$. What is the matrix of T in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$, where $\alpha_1 = (1,0,1)$, $\alpha_2 = \{-1,2,1)$, $\alpha_3 = (2,l,l)$.

3. Find the Galois group of the equation $x^3 - 2 = 0$ over the field Q of rational numbers.

4. Define field extension. If L is a finite extension of K and if K is a finite extension of F, then L is a finite extension of F and [L : F] = [L : K] [K : F].

5. Let H and N be two subgroups of G such that N is normal in G. Then prove that $H \cap N$ is normal subgroup of H and $H/H \cap N \cong HN/N$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Ten (10) marks each. Learners are required to answer any Four (04) questions only. (4×10=40)

1. Prove that every abelian group G is a module over the ring of integers I.

2. For any two vector u and v in an inner product space V prove that:

(a) $\| u + v \| \leq \| u \| + \| v \|$

(b) $\| u \| - \| v \| \leq \| u - v \|$

3. Let G_1 and G_2 be two groups. Let $G = G_1 \times G_2$

$H_1 = \{(a_1\ e_2)\ a \in G_1\} = G_1 \times \{e_2\}$

$H_2 = \{(e_1, b), b \in G_2\} = \{e_1\} \times G_2$

Where e_1 and e_2 are identity elements of G_1 and G_2 respectively. Then prove that G is an internal direct product of H_1 and H_2.

4. Prove that every finite group G has a composition series.

5. Prove that every ring of polynomials $f(x)$ over a field F is a Euclidean ring.

6. Let $T : V \rightarrow W$ be a linear transformation. Then prove the following :

(a) Ker (T) is a vector subspace of V

(b) Range (T) is vector subspace of W

7. If c is a characterstic vector of T, then c can not correspond to more than one characterstic value of T.

8. Let V and W be inner product spaces. Then prove that a linear transformation $T : V \rightarrow W$ is orthogonal if and only if $\| T(u) \| = \| u \|$ for all $u \in V$.