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MAMT-10

Mathematical Programming

MA/M.Sc. Mathematics (MAMT/MSCMT-19)

Second Year Examination, 2021 (Winter)

Time : 2 Hours] **Max. Marks : 80**

Note : This paper is of Eighty (80) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Twenty (20) marks each. Learners are required to answer any Two (02) questions only.

(2×20=40)

1. Using bounded variable technique, solve the following l.p.p

Max. $z = x_1 + 3x_2$

s.t. $x_1 + x_2 + x_3 \leq 10$

$x_1 - 2x_3 \geq 0$
$2x_2 - x_3 \leq 10$
$0 \leq x_1 \leq 8,\ 0 \leq x_2 \leq 4,\ x_3 \geq 0.$

2. Solve the following nonlinear programming problem using the method of Lagrangian multipliers :

Minimize $f(X) = x_1^2 + x_2^2 + x_3^2$
s.t. $4x_1 + x_2^2 + 2x_3 = 14$
$x_1, x_2, x_3 \geq 0.$

3. Solve the following quadratic programming problem using Wolfe's method

Minimize $f(x_1, x_2) = x_1^2 - x_1x_2 + 2x_2^2 - x_1 - x_2$
subject to $2x_1 + x_2 \leq 1$
$x_1, x_2 \geq 0.$

4. Use dynamic programming to solve the following L.P.P.:

Max. $Z = 2x_1 + 5x_2$
subject to $2x_1 + x_2 \leq 43$
$2x_2 \leq 46$
and $x_1, x_2 \geq 0$

5. Define with examples :

(a) Closed and Open set.

(b) Convex set.

(c) Extreme point.

(d) Supporting Hyperplane.

(e) Quadratic form.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Ten (10) marks each. Learners are required to answer any Four (04) questions only. (4×10=40)

1. Prove that $f(x) = \frac{1}{x}$ is strictly convex for $x > 0$ and strictly concave for $x < 0$.

2. Explain :

(a) Integer Programming Problem (I.P.P.).

(b) Mixed Integer Programming Problem.

(c) Fractional Cut.

(d) Cutting plane method.

3. Solve the following I.P.P by branch and bound technique

Max. $Z = x_1 + x_2$

s.t. $3x_1 + 2x_2 \leq 12$

$x_2 \leq 2$

$x_1, x_2 \geq 0$ and integers.

4. Determine the sign of definiteness for each of the following matrices.

(a) $\begin{bmatrix} 2 & 1 & 4 \\ 6 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 & 2 \\ 1 & -3 & 3 \\ 2 & 0 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 & 2 \\ 0 & -3 & 3 \\ 0 & 0 & -5 \end{bmatrix}$

(d) $\begin{bmatrix} -5 & 0 & 2 \\ 4 & -1 & 3 \\ 2 & 5 & -5 \end{bmatrix}$

5. Use Kuhn-Tucker condition to solve the following non-linear programming problem :

Minimize $f(x) = 8x - x^2$

subject to $x \leq 3$

$x \geq 0$

6. Derive the dual of the quadratic programming problem :

$$\text{Min } f(X) = C^T X + \frac{1}{2} X^T GX$$

Subject to $AX \geq b$

where A is $m \times n$ real matrix and G is an $n \times n$ real postive semidefinite symmetric matrix.

7. Prove that every local maximum of the general convex programming problem is its global maximum.

8. Define :

(a) Dynamic programming.

(b) Bellman's Principal of Optimality.

(c) Stage.

(d) State.