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MAMT-09

Integral Transforms and Integral Equations

MA/M.Sc. Mathematics (MAMT/MSCMT-19)

Second Year Examination, 2021 (Winter)

Time : 2 Hours] **Max. Marks : 80**

Note : This paper is of Eighty (80) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Twenty (20) marks each. Learners are required to answer any Two (02) questions only.

(2×20=40)

1. State and prove the Parseval's identity for Fourier transform.

2. Define Mellin transform. If $M\{f(x); p\} = F(p)$, then find the following :

(a) $M\{f(ax); p\}$

(b) $M\{x^a f(x); p\}$

(c) $M\{f(x^a); p\}$

(d) $M\left\{\frac{1}{x} f\left(\frac{1}{x}\right); p\right\}$

(e) $M\{\log x\, f(x); p\}$

3. Solve $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$, $x > 0$, $t > 0$ subject to conditions :

(a) $U(0, t) = 0$

(b) $U = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ where $t = 0$

(c) $U(x, t)$ is bounded.

4. Find the Eigen values and there corresponding Eigen function of the homogeneous Fredholm integral equation of the second kind $g(x) = \lambda \int_0^{2\pi} \sin(x + t)\, g(t)\, dt$.

5. Prove that $L[J_0(t); p] = \frac{1}{\sqrt{1+p^2}}$ and hence deduce that

(a) $L[J_0(at); p] = \frac{1}{\sqrt{p^2 + a^2}}$

(b) $L[tJ_0(at); p] = \frac{p}{(p^2 + a^2)^{3/2}}$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Ten (10) marks each. Learners are required to answer any Four (04) questions only. (4×10=40)

1. If $\bar{f}(p)$ is the Laplace transform of $f(t)$ i.e., $L[f(T); p] = \bar{f}(p)$, then prove the following property.

(a) $L[f(at); p] = \frac{1}{a}\bar{f}\left(\frac{p}{a}\right)$.

(b) $L[e^{at}f(t); p] = \bar{f}(p - a)$.

2. Apply convolution theorem to prove that

$$B(m, n) = \int_0^1 u^{m-1}(1-u)^{n-1}\, du = \frac{\Gamma m\, \Gamma n}{\Gamma(m+n)}, \; (m > 0, n > 0).$$

Hence deduce that

$$\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta = \frac{1}{2} B(m, n) = \frac{\Gamma m \Gamma n}{2\Gamma(m+n)}$$

where B(m, n) is called Beta function.

3. Using Laplace transformation solve the following differential equation $(D^2 + 1)y = t \cos 2t$, $y = 0$, $\frac{dy}{dt} = 0$ when $t = 0$.

4. Find the relation between Fourier transform and Laplace transform.

5. Define the Hankel transform. If $F_v(p)$ and $G_v(p)$ are the Hanel transform of the function $f(x)$ and $g(x)$ respectively, then prove that $\int_0^\infty xf(x)g(x)dx = \int_0^\infty p\, F_v(p) G_v(p) dp$.

6. Show that the function $g(x) = xe^x$ is a solution of the Volterra integral equation $g(x) = \sin x + 2\int_0^x \cos(x-t)\, g(t)dt$.

7. Find the resolvent kernel of the kernel

$K(x, t) = e^{(x+t)}$, $a = 0$, $b = 1$.

8. Find the Laplace transform of $e^{-2t}(3 \cos 6t - 5 \sin 6t)$.
