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MAMT-06

Analysis and Advanced Calculus

MA/M.Sc. Mathematics (MAMT/MSCMT-19)

2nd Year Examination, 2021 (Winter)

Time : 2 Hours] **Max. Marks : 80**

Note : This paper is of Eighty (80) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Twenty (20) marks each. Learners are required to answer any Two (02) questions only.

(2×20=40)

1. Prove that every normed linear space is a metric space. If N be a normed linear space and $x, y \in$ N, then prove that

$$| \|x\| - \|y\| | \leq \|x - y\|$$

2. Define orthonormal set in Hilbert space. If $\{e_1, e_2, ..., e_n)$ be finite orthonormal set in a Hilbert space H, and x be any vector in H, then

(a) $\sum_{i=1}^{n} |(x, e_i)|^2 \leq | x |^2$

(b) $x - \sum_{i=1}^{n} (x, e_i)e_i \perp e_j \forall j$

3. Define the Adjoint operator in Hilbert space. Prove that if T be an operator on a Hilbert space H, then $\exists$ a unique linear operator T^* on H such that

$$(Tx, y) = (x, T^*y) \forall x, y \in H$$

Where T^* is the adjoint operator H.

4. State and proof the Minkowski's Inequality.

5. If B and B' are Banach spaces and T is a linear transformation of B into B', then T is continuous $\Leftrightarrow$ its graph is closed.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Ten (10) marks each. Learners are required to answer any Four (04) questions only. (4×10=40)

1. Every convergent sequence in a normed linear space is a Cauchy sequence.

2. The inner product space in a Hilbert space is jointly continuous *i.e.*, if $x_n \to x$ and $y_n \to y$, then $(x_n, y_n) \to (x, y)$ as $n \to \infty$.

3. If X is an inner product space, then $\| x \| = (x, x)^{1/2}$ is a norm on X.

4. Define compactness in normed space. Prove that every compact subset of a normed linear space is complete.

5. Define the directional derivative. Suppose that X and Y be Banach spaces over the same field K of scalars and V be an open subset of X. Let $f: \mathrm{V} \to \mathrm{Y}$ is differentiability at $x \in \mathrm{V}$. Then show that all the directional derivatives of f exist at x and

$$D_v f(x) = Df(x).\, v \text{ where } v \in \mathrm{V} \text{ is a unit vector.}$$

6. Define orthogonal projection. If P and Q are projections on closed linear subspaces M and N of a Hilbert space H, then $\mathrm{M} \perp \mathrm{N} \Leftrightarrow \mathrm{PQ} = 0 \Leftrightarrow \mathrm{QP} = 0$.

7. If T_1 and T_2 are normal operators on H with the property that either commute with adjoint of the other, then $T_1 + T_2$ and T_1T_2 are normal.

8. Let M be a linear subspace of Hilbert space H. Then M is closed if and only if $\mathrm{M} = \mathrm{M}^{\perp\perp}$.