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MT-609

Integral Equations

MA/MSC Mathematics (MAMT/MSCMT)

4th Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. Show that the function $g(x) = \dfrac{x}{\left(1+x^2\right)^{5/2}}$ is a solution of the integral equation

$$g(x) = \frac{3x+2x^3}{3\left(1+x^2\right)^2} - \int_0^x \frac{3x+2x^3-t}{\left(1+x^2\right)^2} g(t)dt.$$

2. Find the eigen values and eigen function of the homogeneous integral equation

$$g(x) = \lambda \int_0^{\pi} (\cos^2 x \cos 2t + \cos 3x \cos^3 t) g(t) dt$$

3. Solve the following integral equations :

(a) $g(x) = \sin x + \lambda \int_4^{10} x g(t) dt.$

(b) $g(x) = 1 + \lambda \int_0^{\pi} \sin(x + t) g(t) dt.$

4. Solve the integral equation :

$$g(x) = (1 - 2x - 4x^2) + \int_0^x \left\{3 + 6(x - t) - 4(x - t)^2\right\} g(t) dt.$$

5. Prove that the solution of the Fredholm Integral Equation

$$g(x) = f(x) + \lambda \int_a^b R(x, s; \lambda) f(t) ds.$$

is unique for any λ provided that $D(\lambda) \neq 0$.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Show that the function $g(x) = 1 - x$ is a solution of the equation $\int_0^x e^{x-t} g(t)dt = x.$

2. Find the eigen values and eigen functions of the following homogeneous integral equation and solve it

$$g(x) = \lambda \int_0^1 e^x \cdot e^t g(t)dt.$$

3. Convert the following differential equations into integral equations :

(a) $\dfrac{d^2 y}{dx^2} + y = 0;$ $y(0) = 0 = y'(0)$

(b) $\dfrac{d^2 y}{dx^2} + \lambda xy = F(x);$ $y(0) = 1;\ y'(0) = 0$

4. Solve the integral equation by the method of resolvent kernal

$$g(x) = f(x) + \lambda \int_0^1 e^{x-t} g(t)dt$$

5. Define following with example :

(a) Singular integral.

(b) Convolution integral.

(c) Fredholm integral equation of first and second kind.

(d) Volterra integral equation of first and second kind.

6. Solve the homogeneous Fredholm integral equation

$$g(x) = \lambda \int_0^1 e^x . e^t \ \phi(t) dt.$$

7. Solve the following integro-differential equation :

$$g'(x) = \sin x + \int_0^x g(x-t) \cos t \ dt,$$

where $g(0) = 0$.

8. Solve the following integral equation by the method of successive approximations to third order

$$g(x) = 1 + \lambda \int_0^1 (x+t) g(t) \ dt, \text{ by taking } g_0(x) = 1.$$