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MT-606

Analysis and Advanced Calculus-II

MA/MSC Mathematics (MAMT/MSCMT)

4th Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Prove that If H be a given Hilbert space and T^* be adjoint of the operator T. Then T^* is a bounded linear transformation and T determines T^* uniquely.

2. Prove that If P is a projection on a Hilbert space H with range M and null space N, then M $\perp$ N iff P is self adjoint, and in this case N = $M^{\perp}$.

3. Prove that If T is a normal operator on Hilbert space H then each eigenspace of T reduces T.

4. State and prove Inverse function Theorem.

5. Prove that Let f be C^1 map on a compact interval $[a,b]$ into a compact interval $[c,d]$ of R and let g be a continuous function on $[c,d]$ into a Banach space X over K. Then

$$\int_{a}^{b} (D(f(s))g(f(s))ds = \int_{f(a)}^{f(b)} g(t)c.$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Prove that If T is an arbitrary operator on Hilbert space H, then T = 0 iff $(Tx, y) = 0 \forall x \varepsilon H$ iff T = 0.

2. Prove that if X and Y be two Banch spaces over the same field K. In the set of all functions tangential to a function f at $v \in$ V, there is at most one function $\varnothing$: X $\rightarrow$ Y, of the form $\varnothing(x) = f(v) + g(x - v)$, where g: X $\rightarrow$ Y is linear, where V is an non-empty open subset of X.

3. Define Adjoint Operator, Self-Adjoint Operator and Normal operator.

4. Define Lipschitz's Property and Existence Theorem for differential equation.

5. State Taylor's Theorem, Taylor's formula with Lagrange's Reminder and Taylor's Formula with Integral Reminder.

6. Define derivative of a map in Banach spaces and show that derivative of the constant function is the zero linear map and derivative of a continuous linear mapping is the mapping itself.

7. Prove that Let f be a regulated function on a compact interval $[a, b]$ of R into a Banach space X then at each $t \in [a, b]$, the function F: $[a, b] \rightarrow$ X, $F(t) = \int_0^t f$, $t \in [a, b]$ is continuous.

8. Define eigen values, eigen vectors and Projection on a Hilbert Space.
