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MT-604

Integral Transforms

M.A./M.Sc. Mathematics (MAMT/MSCMT)

3rd Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Find $L^{-1}\left[\frac{\cosh u\sqrt{P}}{P\cosh\sqrt{P}}\right]$, where $0 < u < 1$.

2. Solve $t\,y'' + (t-1)y' - y = 0$, $y(0) = 5$, $y(\infty) = 0$.

3. State and Prove Convolution Theorem for Fourier Transforms.

4. Solve $\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$, $x > 0$, $t > 0$ subject to conditions :

(a) $U(0, t) = 0$.

(b) $U = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ when $t = 0$.

(c) $U(x, t)$ is bounded.

5. Define :

(a) Laguerre polynomial.

(b) Hypergeometric Function.

(c) Beta Function.

(d) Bessel Function.

SECTION–B
(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Find the Laplace transform of :

(a) $e^{-2t} - \sin 5t + 4 \cos 7t + 9t^3 - 5$.

(b) $\cosh^2 4t$.

2. Find fourier transform of $f(t)$ defined by $f(t) = \begin{cases} 1, & |t| < a \\ 0, & |t| \geq a \end{cases}$

and hence prove that $\int_0^\infty \frac{\sin^2 at}{t^2}\, dt = \frac{\pi a}{2}$.

3. Find the Mellin Transform of sin x and show that

$$M^{-1}\left\{\Gamma(p)\sin\left(\frac{p\pi}{2}\right) f^*(1-p);\, x\right\} = \sqrt{\frac{\pi}{2}}\, F_s\{f(t);\, x\},$$

where $f^*(p) = M(f(t);\, p)$.

4. Find the Hankel transform of $\frac{\sin ax}{x}$ taking $xJ_0\,(px)$ as the kernel.

5. If $L^{-1}\left\{\frac{p^2-1}{(p^2+1)^2};\, t\right\} = t \cos t$, then find $L^{-1}\left\{\frac{9p^2-1}{9p^2+1)^2};\, t\right\}$.

6. Solve $(D^2 + 1)y = t^2 \cos 2t$, $y = 0$, $dy/dt = 0$ when $t = 0$.

7. Prove that

$$H_v\{x^v(a^2 - x^2)^{\mu-v-1}U(a - x);p\}$$
$$= 2^{\mu-v-1}\Gamma(\mu - v)\, p^{v-\mu}\, a^{\mu} J_{\mu}(pa),\ a > 0,\ \mu > v$$

8. If the flow of heat is linear so that the variation of θ (temperature) with z and y-axes may be neglected and if it is assumed that no heat is generated in the medium, then solve the differential equation $\frac{\partial \theta}{\partial t} = k \frac{\partial^2 \theta}{\partial x^2}$, where $-\infty < x < \infty$ and $\theta = f(x)$ when $t = 0$, $f(x)$ being a given function of x.
