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MT-601

Analysis and Advanced Calculus-I

MA/MSC Mathematics (MAMT/MSCMT)

3rd Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Let p be a real number such that $1 \leq p < \infty$. Show that the space l_p^n of all n-tuples of scalars with the norm defined by

$\|x\|_p = \left\{ \sum_{i=1}^{n} |x_i|^p \right\}^{\frac{1}{p}}$ is a Banach space.

2. Define (i) Inner product spce (ii) Hilbert space and give an example.

3. Give an example of an inner product space which is not a Hilbert space.

4. State and prove BessePs inequality in Hilbert space.

5. State and prove of Open mapping Theorem.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Prove that every convergent sequence in a normed linear space is a Cauchy sequence.

2. Prove that on a finite dimensional linear space X, all norms are equivalent.

3. Prove that if a normed space N is reflexive, it is complete. -

4. Prove that every Hilbert space is reflexive.

5. Check that the space l^p with $p \neq 2$ is an inner product space or not.

6. State and prove Polarisation Identity.

7. Let M be a linear subspace of Hiibert space H. Then M is closed if and only if $M = M^{\perp\perp}$.

8. Define an orthonormal set and complete orthonormal set with example.
