S-77

Total Pages : 4 Roll No. .....

MT-510

Mechanics-II

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. If initially the axis of the top is horizontal and it is set spinning with angular velocity w in a horizontal plane, prove that the axis will start to rise if $n\mathrm{C}w > mgh$ and that, when $n\mathrm{C}w = 2mgh$, the axis will be rise to an angular distance $\cos^{-1}\left(\frac{\mathrm{A}w}{nc}\right)$, provided that $\mathrm{A}w < n\mathrm{C}$, and will there be instantaneous rest. A, C and n have their usual meanings.

2. State and prove the principle of least action for a conservation holonomic system.

3. The velocity field at a point in fluid is given as $\vec{q} = \frac{x}{t}\hat{i} + y\hat{j} + 0.\hat{k}$, obtain path lines.

4. Find the equation of continuity in Lagrange's method. Show that it is equivalent to $\frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0.$

5. State and prove Bernoulli's theorem.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Show that $u = \frac{-2xyz}{(x^2 + y^2)^2}$, $v = \frac{(x^2 - y^2)z}{(x^2 + y^2)^2}$ and

$w = \frac{y}{(x^2 + y^2)}$ are the velocity components of a possible fluid motion. Is this motion irrotational ?

2. A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is

$$\frac{\partial p}{\partial t} + \frac{\partial(\rho w)}{\partial \theta} = 0.$$

3. Show that the ellipsoid

$$\frac{x^2}{a^2 k^2 t^{2n}} + kt^n \left(\frac{y^2}{b^2} + \frac{z^2}{c^2} \right) = 1$$

is a possible form of the boundary surface of a liquid.

4. Liquid is contained between two parallel planes, the surface is a circular cylinder of radius a whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius b is suddenly annihilated. Prove that if π be the pressure at the outer surface, the initial pressure at any point on the liquid distance r from the centre is

$$\pi \frac{\log r - \log b}{\log a - \log b}.$$

5. What arrangement of sources and sinks will give rise to the function $w = m \log \frac{(z^2 - a)}{z}$? Draw a rough sketch of a stream line. Prove that two of the stream lines sub divide into the circle $r = 0$ and the axis of y.

6. To determine the image of the source with respect to a circle.

7. In irrotational motion in two dimension, prove that

$$\left(\frac{\partial q}{\partial x}\right)^2 + \left(\frac{\partial q}{\partial y}\right)^2 = q\nabla^2 q.$$

8. Define the following :

(a) Velocity potential.

(b) Doublet.
