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MT-509

Differential Geometry and Tensor-II

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Prove that there is no distinction between contravariant and Co-variant tensors when we restrict ourselves to rectangular Cartesian transformation of Co-ordinates.

2. Prove that christoffel symbols vanish identically if and only if g_{ij}'s are constants.

3. Prove that the curves of the family $\frac{V^3}{u^3}$ = Constant are geodesies on a surface with metric.

$V^2 \, du^2 - 2uvdudv + 2u^2 \, dv^2; \; u > o, \; v > o.$

4. Prove that a necessary and sufficient condition that a curve on a developable surface be a geodesic is that the surface be the rectifying developable of the curve.

5. State and prove Gauss-Bonnet theorem.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. If A_i is a covariant vector, then prove that $\left(\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}\right)$ is a covariant tensor of rank 2.

2. Prove that $\delta_j^i a^{jk} = a^{ik}$.

3. If λ_i and μ^i are the components of a covariant and contravariant vector respectively, then prove that the Sum $\lambda_i \mu^i$ is an invariant.

4. Prove that the magnitude of a vector is an invariant.

5. A particle is constrained to move on a smooth surface under no forces except the normal reaction. Show that its path is a geodesic.

6. If a geodesic on a surface of revolution cuts the meridians at a constant angle, then prove that the surface is a right cylinder.

7. Write a short note on 'Bonnet's formula' for geodesic curvature.

8. Prove that on a surface of negative curvature, two geodesies cannot meet in two points and enclose a simply connected area.
