S-75

Total Pages : 3   Roll No. ....

MT-508

Special Functions

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2022 (Dec.)

Time : 2 Hours]   **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

$(2 \times 9\frac{1}{2} = 19)$

1. Define :

(a) Ordinary and Singular point.

(b) Regular Singular point.

(c) Radius of convergence.

(d) Irregular singular point.

2. Prove that $J_n(x) = (-2)^n x^n \dfrac{d^n}{d(x^2)^n}.J_0(x)$.

3. Discuss differentiation and integral representations for the Hypergeometric Junction.

4. Prove that $n[P_n Q_{n-1} - Q_n P_{n-1}] = 1$.

5. Show that the Hermite polynomials are orthogonal over $(-\infty, \infty)$ with respect to the weight function e^{-x^2}.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only.   (4×4=16)

1. Define :

(a) Hevmite polynomials.

(b) Bessel Functions.

2. Show that Hypergeometric function does not change if the parameters α and β are interchanged, keeping γ fixed.

3. Show that $J_{-n}(x) = (-1)^n J_n(x)$.

4. Show that $L'_n(0) = -n$.

5. Prove that $H_5(x) = 32x^5 - 160x^3 + 120x$.

6. Show that $L_n(x) = \dfrac{e^x}{n!} \dfrac{d^n}{dx^n}(x^n e^{-x})$.

7. Prove that $H_{2n+1}(0) = 0$.

8. Prove that $J_{-1/2}(x) = \sqrt{\left(\dfrac{2}{\pi x}\right)} \cos x$.

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