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MT-507

Topology

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only.

(2×9½=19)

1. Let $\tau = \{\phi, X, \{1\}, \{1,2\}, \{1,2,5\}, \{1,2,3,4\}, \{1,3,4\}\}$ be the topology on $X = \{1,2,3,4,5\}$. Determine limit points, closure, interior, exterior and boundary point of the set $A = \{3,4,5\}$.

2. Show that characteristic function of $A \subset X$ is continuous on X iff A is both open and closed in X.

3. Prove that a topological space (X, τ) is a normal space if and only if for any closed set F and an open set G containing F, there exist an open set V such that $F \subset V \subset \overline{V} \subset G$, here $\overline{V}$ is closure of V.

4. Prove that two disjoint sets A and B are separated in a topological space (X, τ) iff they are both open and closed in the subspace $A \cup B$

5. Prove that a subset A of R is compact if and only if A is bounded and closed.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Let (X, τ) be a topological space and let A, B be non-empty subset of X, then prove that $(A \cup B') = A' \cup B'$, where A' = derived set of A.

2. Prove that a function $f : X \to Y$ is continuous iff the inverse image of each member of a base B for Y is an open subset of X.

3. Prove that a topological space (X, τ) is a T_1 – space iff $\{x\}$ is closed $\forall x \in X$.

4. Prove that T_4 – space is T_3 – space.

5. Prove that every closed subspace of a normal space is normal.

6. Prove that continuous image of compact space is compact.

7. Let $X = \{a,b,c,d\}$ and $\tau = \{\phi, X,\{b\},\{b, c\},\{b,c,d\}\}$, then show that X is connected.

8. Prove that a topological space X is disconnected iff X is the union of two non-empty disjoint open sets.