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# MT-506

## Advanced Algebra-II

MA/MSC Mathematics (MAMT/MSCMT)

2nd Semester Examination, 2022 (Dec.)

**Time : 2 Hours]** **[Max. Marks : 35**

**Note :** This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

## SECTION–A
## (Long Answer Type Questions)

**Note :** Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

**1.** Let $t$: $R^3 \to R^3$ be a linear transformation such that $t(a, b, c) = (a + c, -2a + b, -a + 2b + c)$. What is the matrix of $t$ in ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$, where $\alpha_1 = (1,0,1)$, $\alpha_2 = \{-1,1,1\}$ $\alpha_3 = \{0,1,1\}$.

2. An $n \times n$ square matrix A over a field F is invertible iff det (A) $\neq$ 0 and $\det(A^{-1}) = \frac{1}{\det(A)}$.

3. (a) Define

   (i) orthogonal set.

   (ii) orthonormal basis.

   (iii) orthonormal set.

   (b) State and Prove schwarz inequality in inner product space.

4. (a) Define Galois group and galois extension.

   (b) Prove that the order of the Galois group G(K|F) is equal to the degree of K over F. i.e. o[G(K|F)] = [K:F].

5. Let V and V 'be inner-product spaces. Then a linear transformation $t : V \to V'$ is orthogonal if and only if $\| t(u) \| = \| u \|$ for all $u \in V$.

## SECTION–B

## (Short Answer Type Questions)

**Note :** Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Let H be a subgroup of all automorphism of a field K. Then prove that the fixed field of H is a subfield of K.

2. Prove that the order of the Galois group $G(K|F)$ is equal to the degree of K over F. i.e. $o\{G(K|F)\} = [K : F]$.

3. Prove that for any matrix A over a field F, rank (A) = rank $(A^T)$.

4. Prove that the similar matrices have the same characteristic polynomial and hence the same eigen values.

5. Prove that if $u = (a_1, a_2)$, $v = (b_1, b_2) \in R^2$ then $< u, v \geq a_1 b_1 - a_2 b_1 - a_1 b_2 + 4a_2 b_2$ defines an inner product.

6. If W is a subspace of an inner product space $R^3$ spanned by $B_1 = \{(1,0,1). (1,2, -2)$, then find a basis of orthogonal complement W.

7. Let V and V' be inner product spaces. Then every orthogonal linear transformation $t : V \to V'$ is a monomorphism of vector spaces.

8. Define :

   (a) Separable extensions.

   (b) Normal extension.