S-70

Total Pages : 3 Roll No. ....

MT-503

Differential Equation and Calculus of Variation

MA/MSC Mathematics (MAMT/MSCMT)

1st Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Solve $r + (a + b)s + abt = xy$ by Monge's method.

2. Use the method of separation of variables to solve the PDE.

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0.$$

3. Reduce the equation $(n-1)^2 \frac{\partial^2 u}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = ny^{2n-1} \frac{\partial z}{\partial y}$ to canonical form and find its general solution.

4. Find the eigenvalues and eigenfunction for the boundary value problem $y'' - 3y' + 2(1 + \lambda)y = 0$; $y(0) = 0$, $y(1) = 0$.

5. Find the extremal of the functional

$$I = \int_0^1 (1 + y''^2)\, dx$$

under the conditions $y(0) = 0$, $y'(0) = 1$, $y(1) = 1$, $y'(1) = 1$.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Show that there are two values of the constant for which $\frac{k}{x}$ is an integral of $x^2(y_1 + y^2) = 2$, and hence obtain the general solution.

2. Solve $yz(y + z)dx + zx(x + z)dy - xy(x + y)dz = 0$

3. Find the characteristics of $y^2 r - x^2 t = 0$.

4. Solve $2r + (p + x)S + yt + y(rt - s^2) + q = 0$.

5. Test for extremum of the functional

$$F(y(x)) = \int_0^1 [x^2y^2 + x^2y'] \, dx, \; y(0) = 0, \; y(1) = 1.$$

6. Solve $-t = \frac{x}{y^2}$.

7. Show that the equation $\frac{\partial^2 z}{\partial x^2} + 2x \frac{\partial^2 z}{\partial x \partial y} + (1 - y)^2 \frac{\partial^2 z}{\partial y^2} = 0$ is elliptic for values of x and in the region $x^2 + y^2 < 1$, parabolic on the boundary and hyperbolic outside this region.

8. Prove that eigenvalues of Sturm-Liouville system are real.
