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Total Pages : 3 Roll No. ....

MT-502

Real Analysis

MA/MSC Mathematics (MAMT/MSCMT)

1st Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. If E_n is monotonic non-increasing sequence of measurable sets, then show that the limit $E = \bigcap_{k=1}^{\infty} E_k$ is a measurable set.

2. Show that not every measurable set is a Borel set.

3. State and prove Egoroff's theorem.

4. Let f_n be a sequence of measurable functions on a measurable set E. Let

$$|f_n(x)| < M \ \forall n \in \mathbb{N} \text{ and } \forall x \in E,$$

where M is a fixed positive constant. If $f(x) = \lim_{n\to\infty} f_n(x)$ for each $x \in$ E, then show that

$$\lim_{n\to\infty} \int_E f_n(x)dx = \int_E f(x)dx.$$

5. Show that the normed spaces L^p, $1 \leq p \leq \infty$, are complete.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Let $E_1, E_2, \ldots$ be a sequence of pairwise disjoint subsets of real numbers. Show that

$\mu_*(E_1 + E_2 + \ldots) \geq \mu_*(E_1) + \mu_*(E_2) + \ldots,$

where $\mu_*(E)$ denotes the Lebesgue inner measure of E.

2. Is the set of all irrational numbers in the interval [0,1] measurable? If not, give a justification. If yes, find its measure with a proper justification.

3. Show that the characteristic function of a set E is measurable if and only if E is a measurable set.

4. Show that a step function is a measurable function.

5. Give an example of a function which is not integrable in the sense of Lebesgue.

6. Show that a set E is outer measurable if and only if its complement E^c is outer measurable.

7. Show that the function $f : [0,1] \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{1}{x}; & 0 < x < 1 \\ 5\,; & x = 0 \\ 7\,; & x = 1 \end{cases}$$

is a measurable function.

8. If $f \in L^p$, and $g \leq f$, then show that $g \in L^p$.
