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Total Pages : 3 Roll No. ....

MT-501

Advanced Algebra-I

MA/MSC Mathematics (MAMTVMSCMT)

1st Semester Examination, 2022 (Dec.)

Time : 2 Hours] **[Max. Marks : 35**

Note : This paper is of Thirty Five (35) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nine and Half (9½) marks each. Learners are required to answer any Two (02) questions only. (2×9½=19)

1. Let G_1 and G_2 is a group. Prove that $G_1 \times G_2$ is a group.

2. Let R be a ring and M be an M-module. Then

(a) ro = 0 for all $r \in$ R.

(b) om = 0 for all m $\in$ M.

(c) R($m - n$) = $rm - rn$ for all $r \in$ R $m, n \in$ M.

3. Let $t : V \to V'$ be a linear transformation. Then

(a) Ker(t) is a vector subspace of V.

(b) lm(t) is a vector subspace of V$'$.

4. If K is a finite field extension of a field F and L is a finite field extension of K, then L is a finite field extension of F and [L : F] = [L : K] [K : F].

5. Prove that the ring of Gaussian integer is a Euclidean ring.

SECTION–B
(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Four (04) marks each. Learners are required to answer any Four (04) questions only. (4×4=16)

1. Define (a) Group Homomorphism (b) Kernal of homomorphism.

2. Prove that any two conjugate classes of a group are either disjoint or identical.

3. Prove that every finite abelian group is solvable.

4. Define (a) Divisor (b) Prime element.

5. Prove that every ring of polynomial F [x] over a field is a Euclidean ring.

6. Show that the following mapping is linear

$t : R^3 \rightarrow R^2$ given by $t(x, y, z) = (z, x + y)$ for all $(x, y, z) \in R^3$.

7. Let Q be the field of rational numbers, then show that

$$Q\left(\sqrt{2}, \sqrt{3}\right) = Q\left(\sqrt{2} + \sqrt{3}\right).$$

8. Let G_1 and G_2 be groups, then $G_1 \times G_2 \cong G_2 \times G_1$.
