S-1057

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MAMT-09

Integral Transforms and Integral Equations

MA/M.Sc. Mathematics (MAMT/MSCMT)

2nd Year Examination, 2022 (Dec.)

Time : 2 Hours] **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only. (2×19=38)

1. Find the Fourier sine and cosine transform of $f(t)$, if

$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 2 - t, & 1 < t < 2 \\ 0. & t > 2 \end{cases}$$

2. Prove that the characteristic numbers (eigen values) of a symmetric kernel are real.

3. Find the eigenvalues and there corresponding eigenfunction of the homogeneous integral equation

$$g(x) = \lambda\int_0^{\pi}\left[\cos^2 x\cos 2t + \cos 3x\cos^3 t\right]g(t)dt.$$

4. Evaluate the Laplace transform of the following functions:

(a) $\sin at - at \cos at + \dfrac{\sin t}{t}$

(b) $\sin \sqrt{t}$

(c) $\dfrac{\cos\sqrt{t}}{\sqrt{t}}$

5. Let $F_v(p)$ and $F'_v(p)$ be the Hankel transform of order v of $f(x)$ and $f'(x) = \dfrac{df}{dx}$ respectively. Then $H_v\,\{f'(x); p\} = F'_v(p)$

$$= -\frac{p}{2v}\{(v+1)F_{v-1}(p) - (v-1)F_{v+1}(p)\}$$

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Find the Laplace transform of e^{-1} (3 sin h 2t – 5 cos h 2t).

2. Find the inverse Laplace transform of

$$\frac{p}{\left(p^4 + 4a^4\right)}$$

3. Solve

$(D^2 + 1)y = t \cos 2t,\ y = 0,\ \dfrac{dy}{dt} = 0$, when $t = 0$.

4. Find the solution of

$\dfrac{\partial u}{\partial t} = 3\dfrac{\partial^2 u}{\partial x^2}$, given that $u_x(0, t) = 0$, $u\left(\dfrac{\pi}{2}, t\right) = 0$ and $u(x, 0) = 30 \cos 5x$.

5. Prove that if n is a positive integer,

$$M\left[\left(x\frac{d}{dx}\right)^n f(x); p\right] = (-1)^n p^n F(p),$$

where $M\{f(x);p\} = F(p)$.

6. Find the Hankel transform of $x^{v}e^{-ax}$, taking $xJ_{v}(px)$ as the kernel.

7. Show that the function $g(x) = 1$ is a solution of the Fredholm integral equation $g(x) + \int_0^1 x\,(e^{xt} - 1)g(t)dt = e^{x} - x.$

8. Find the resolvent kernel of the integral equation

$$g(x) = e^{x^2} + {}^{2x} + 2\int_0^x e^{x^2 - t^2}\ g(t)dt.$$