S-1054

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MAMT-06

Analysis and Advanced Calculus

MA/M.Sc. Mathematics (MAMT/MSCMT)

2nd Year Examination, 2022 (Dec.)

Time : 2 Hours] **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A

(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. Prove that every normed linear space is a metric space.

2. State and Prove Hahn Banach theorem.

3. (a) Prove that if x and y are any two orthogonal vectors in a Hilbert space H, then

$\|x + y\|^2 = \|x - y\|^2 = \| x \|^2 + \| y |^2.$

(b) If M is a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$.

4. Let f be a continuous function on a comapact interval $[a, b]$ of R into a Banach space X over K. Let F be the function $t \to \int_a^b f$ on $[a, b]$ into X. Let g be any differentiable function on $[a, b]$ into X such that $Dg = f$. Then F is differentiable, $DF = f$ and $\int_a^b f = F(b) - F(a) = g(b) - g(a)$.

5. If H be a Hilbert space and $\{e_i\}$ be an orthonormal set in H, then prove that the following statement are equivalent

(a) $\{e_i\}$ is complete.

(b) $x \perp \{e_i\} \Rightarrow x = 0$.

(c) If x is an arbitrary vector in H, then $x = \Sigma(x, e_i)e_i$.

(d) If x is an arbitrary vector in H, then $\| x \|^2 = \Sigma|(x, e_i)|^2$.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Every convergent sequence in a normed linear space is a Cauchy sequence.

2. Let N and N' be normed linear spaces over the same scalar field and let T be a linear transformation of N into N'. Then T is bounded if it is continuous.

3. State and prove Closed graph theorem.

4. Define with example :

(a) Inner product space.

(b) Hilbert space.

5. If T is an operator on a Hilbert space H, then T is normal iff its real and imaginary parts commute.

6. If T is normal operator on a Hilbert space H, then eigenspaces of T are pairwise orthogonal.

7. Define :

(a) c-Lipshitz property.

(b) locally Lipschitz.

(c) Regulated function.

8. If T be a linear transformation from a normed linear space N into normed space N', then T is continuous either at every point or at no point of N.