S-1053

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MAMT-02

Real Analysis and Topology

MA/M.Sc. Mathematics (MAMT/MSCMT)

1st Year Examination, 2022 (Dec.)

Time : 2 Hours] **Max. Marks : 70**

Note : This paper is of Seventy (70) marks divided into two (02) Sections A and B. Attempt the questions contained in these sections according to the detailed instructions given therein.

SECTION–A
(Long Answer Type Questions)

Note : Section 'A' contains Five (05) long answer type questions of Nineteen (19) marks each. Learners are required to answer any Two (02) questions only.

(2×19=38)

1. (a) Proved that closed interval [0,1] is uncountable.

(b) If A and B any two disjoint subsets of R, then $m^*(A \cup B) = m^*(A) + m^*(B)$.

2. The space L^p is complete for $p \geq 1$.

3. (a) Prove that a second countable space is always first countable space, but converse is not true.

(b) Define Homeomorphism with example.

(c) Define T_2 space with example.

4. Prove that a topological space X is compact iff every collection of closed subsets of X with the FIP (Finite intersection property) is fixed, that is, has a non-empty intersection.

5. (a) Defined :

(i) Net

(ii) Ultra net

(iii) Filter

(b) Let Y be subset of topological space (X, τ). Prove that the set Y is τ-open iff no net in $(X - Y)$ converges to a point in Y.

SECTION–B

(Short Answer Type Questions)

Note : Section 'B' contains Eight (08) short answer type questions of Eight (08) marks each. Learners are required to answer any Four (04) questions only. (4×8=32)

1. Define :

(a) Denumerable set.

(b) σ – algebra. .

(c) σ – ring.

(d) Borel sets.

2. Let f and g be bounded measurable function on a measurable set E and $f = g$ almost everywhere on E. Then show that

$$\int_E f(x)\ dx = \int_E g(x)\ dx.$$

3. If a function is summable on E, then it is finite almost everywhere on E.

4. Let $f(x) \in L^p$ and $g(x) \in L^p$ where $p \geq 1$, then $\|f + g\|_p \leq \|f\|_p + \|g\|_p$.

5. Define

(a) Topological space.

(b) Closed Set.

(c) Closure of a set.

6. Prove that every subspace of a T_2-space is a T_2-space.

7. Define :

(a) Locally connected space.

(b) Seperated Sets.

(c) Connected sets.

8. The product space $(X \times Y, W)$ is hausdorff if the space (X, τ) and (Y, V) are Hausdorff.