Roll No. ……..

PHY–501

Mathematical Physics and Classical Mechanics

M. Sc. PHYSICS (MSCPHY-12/13/16/17)

First Year, Examination, 2018

Time : 3 Hours **Max. Marks : 80**

Note : This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Learners are required to attempt the questions contained in these Sections according to the detailed instructions given therein.

Section–A

(Long Answer Type Questions)

Note : Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. Describe generating function for Legendre polynomials. Show that :

   (a) $P_n(1) = 1$

   (b) $P_n(-x) = (-1)^n P_n(x)$

   Find $P_n(-1)$.

2. (a) Show that the Fourier transform of a Gaussian function is also Gaussian in the corresponding Fourier space.

(b) Show that Laplace transform possesses the properties of linearity, shifting and change of scale.

3. (a) Explain the Hamilton-Jacobi equation for Hamilton's characteristic function.

(b) Obtain the equation of motion of a dynamical variable F (q, p, t) in terms of the Poisson bracket.

4. (a) State and prove addition and subtraction rules governing tensor analysis.

(b) Construct a backward difference table using the values given below :

x	$y = f(x)$
0	2
1	3
2	5
3	6
4	10

Section–B
(Short Answer Type Questions)

Note : Section 'B' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer *four* (04) questions only.

1. Show that :

$$\int x J_0^2(x)\,dx = \frac{1}{2}x^2\left[J_0^2(x) + J_1^2(x)\right]$$

2. Prove the orthogonality of the Hermite polynomials :

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x)\,dx = 0, m \neq n$$

3. Find the Fourier transform of the function :

$$f(t) = \begin{cases} 0 & t < 0; \\ e^{-\alpha t} & t \geq 0; \end{cases} \quad \alpha > 0$$

4. Find the Laplace transform of the function $t.\, e^{2t}$.
5. What is a metric tensor and express it as a sum of a symmetric and skew symmetric tensors ?
6. Derive Lagrange's equations from D'Alembert's principle.
7. Derive an expression for the Stirling interpolation formula.
8. Explain the basic idea of numerical differentiation. Discuss with *one* suitable example.

Section–C

(Objective Type Questions)

Note : Section 'C' contains ten (10) objective type questions of one (01) mark each.

Choose the correct option :

1. If J_0 and J_1 are Bessel function, then $J_1'(x)$ is given by :

   (a) $-J_0$

   (b) $J_0(x) - \frac{1}{x} J_1(x)$

   (c) $J_0(x) + \frac{1}{x} J_1(x)$

   (d) None of these

2. If $P_n(x)$ be the Legendre polynomial, then $P_n'(-x)$ is equal to :

(a) $(-1)^n P_n(x)$

(b) $(-1)^n P_n'(x)$

(c) $(-1)^{n+1} P_n'(x)$

(d) $P_n''(x)$

3. What is Fourier sine transform of $xe^{-x^2/2}$?

(a) $\frac{s}{\sqrt{2}} e^{-s^2/2}$

(b) se^{-s^2}

(c) $se^{-s^2/2}$

(d) None of these

4. Find the Laplace transform of $F(t) = \begin{cases} t, & 0 < t < 2 \\ 2, & 2 < t \end{cases}$:

(a) $\frac{1 - e^{-2s}}{s}$

(b) $\frac{1 - e^{-2s}}{s^3}$

(c) $\frac{1 - e^{-2s}}{2s^2}$

(d) $\frac{1 - e^{-2s}}{s^2}$

5. If S_{ij} is a symmetric tensor and A_{ij} is antisymmetric tensor, what is the product $A_{ij}S_{ij}$?

   (a) a tensor of mixed symmetry

   (b) an antisymmetric tensor

   (c) a symmetric tensor

   (d) zero

6. For the Ricci tensor $R_{\mu\nu}$, what is the quantity $g^{\mu\nu}R_{\mu\nu}$ in summation convention ?

   (a) tensor of rank four

   (b) tensor of rank two

   (c) any scalar

   (d) scalar curvature

7. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 \right) - V(r)$. The generalized moment corresponding to r and θ are given by :

   (a) $m\dot{r}$ and $mr^2\dot{\theta}$

   (b) $m\dot{r}$ and $mr\dot{\theta}$

   (c) $m\dot{r}^2$ and $mr^2\dot{\theta}$

   (d) $m\dot{r}^2$ and $mr^2\dot{\theta}^2$

8. The action and angle variable have the dimensions of :
   (a) force and angle
   (b) angular momentum and angle
   (c) energy and angle
   (d) are dimensionless quantities

9. What is the order of error in the expression $D^2 = \frac{1}{h^2}\left(\delta^2 - \frac{\delta^4}{12}\right)$?
   (a) $O(h^2)$
   (b) $O(h^3)$
   (c) $O(h^4)$
   (d) $O(h^5)$

10. What is $(\Delta + \nabla)$?
   (a) $\mu\delta$
   (b) $2\delta\mu$
   (c) $E + E^{-1}$
   (d) δ^2