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# MAT–506

## Analysis and Advanced Calculus

M.Sc. MATHEMATICS (MSCMAT-12)

Second Year, Examination, 2018

**Time : 3 Hours** **Max. Marks : 80**

**Note :** This paper is of **eighty (80)** marks containing **three (03)** Sections A, B and C. Attempt the questions contained in these Sections according to the detailed instructions given therein.

### Section–A

### (Long Answer Type Questions)

**Note :** Section 'A' contains four (04) long answer type questions of nineteen (19) marks each. Learners are required to answer *two* (02) questions only.

1. If N be a normed linear space and M a subspace of N, then $\overline{M}$ (Closure of M) is also a subspace of N.
2. State and prove Open Mapping theorem.
3. A closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
4. State and prove inverse function theorem.

## Section–B

## (Short Answer Type Questions)

**Note :** Section 'B' contains eight (08) short answer type questions of eight (08) marks each. Learners are required to answer *four* (04) questions only.

1. If N be a normed linear space and let $x, y \in N$, Then

$$\big\| \|x\| - \|y\| \big\| \le \|x - y\|$$

2. Explain Riesz Lemma.
3. If T is a continuous linear transformation of a normed linear space N into another normed linear spac $N'$ then T is bounded.
4. Define Hilbert space. In a Hilbert space prove :

$$(x, \beta y + yz) = \bar{\beta}(x, y) + \bar{\gamma}(x, z)$$

5. If *x* and *y* are any two vectors in a Hilbert space, then show that :

$$\|x + y\|^2 - \|x - y\|^2 = 4 \operatorname{Re}(x, y)$$

6. If S is a non-empty subset of a Hilbert space H, show that $s^{\perp} = s^{\perp\perp\perp}$.
7. If T is an arbitrary operator on a Hilbert space H, then T = O iff $(Tx, y) = 0$ for all *x* and *y*.
8. If T is an arbitrary operator on a Hilbert space H, and if $\alpha$ and $\beta$ are scalars such that :

$|\alpha| = |\beta|$, then $\alpha T + \beta T^*$ is normal.

## Section–C

## (Objective Type Questions)

**Note :** Section 'C' contains ten (10) objective type questions of one (1) mark each. All the questions of this Section are compulsory.

Write True/False in the following questions :

1. For a normal linear space $\|x\| = 0$ iff $x = 0$.
2. In a normed linear space every sequence is Cauchy sequence.
3. The $L_P$ spaces are not normed linear spaces.
4. A normed linear space is separable if its dual space is separable.
5. Every complex Banach space is a Hilbert space.
6. In a Hilbert space :

$$\left|(x, y) \geq \|x\| \ \|y\|\right|$$

7. If S is non-empty subset of a Hilbert space H, then $s \subset s^{\perp\perp}$.
8. An orthonormal set is complete if it contains an orthonormal set.
9. A Hilbert space is finite dimensional iff every complete orthonormal set is a basis.
10. An operator T on Hilbert space H is self adjoint iff $(T x, x) = 0$ for all $x \in H$.